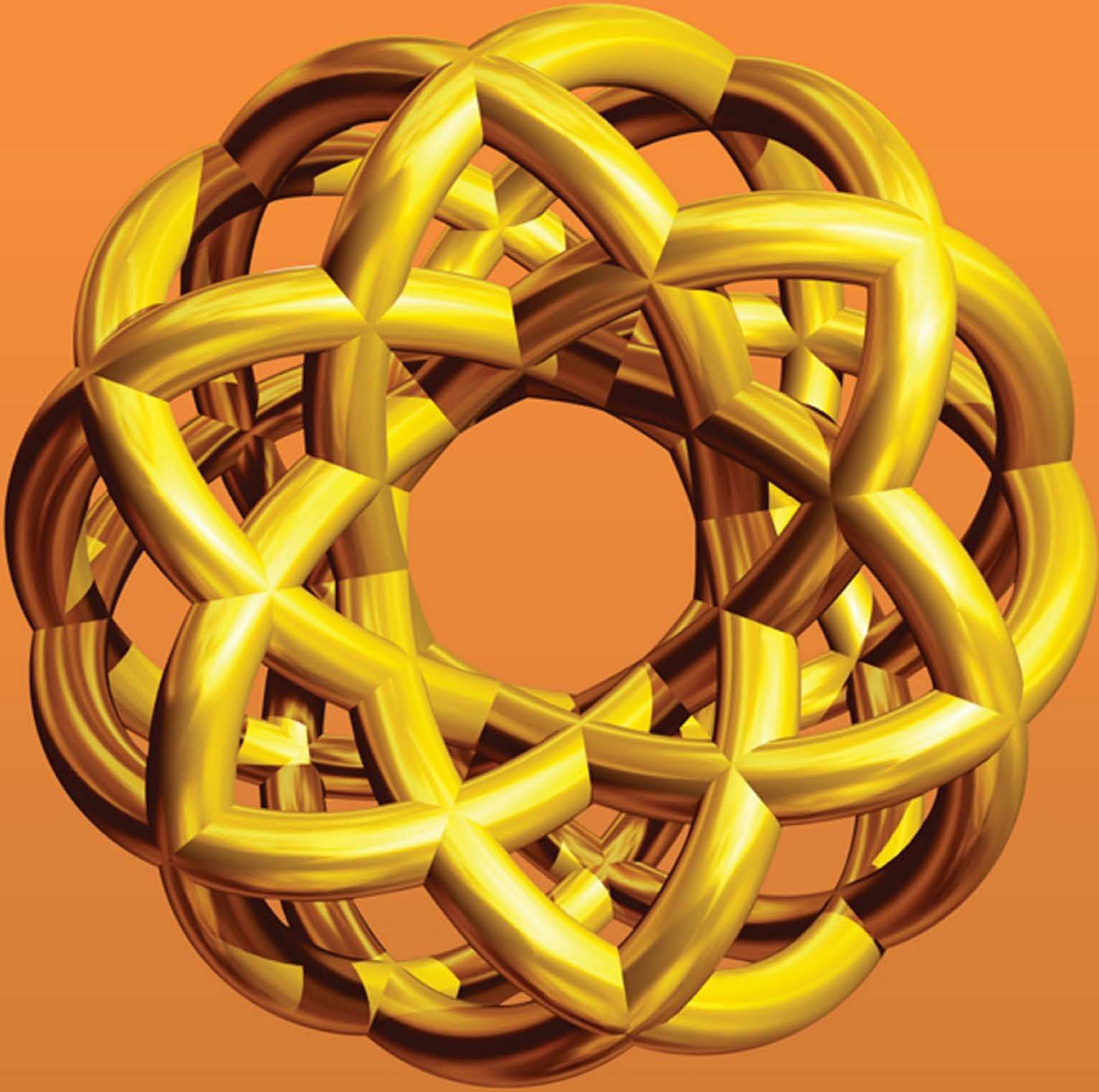
PRECALCULUS
A CONCISE COURSE

3e



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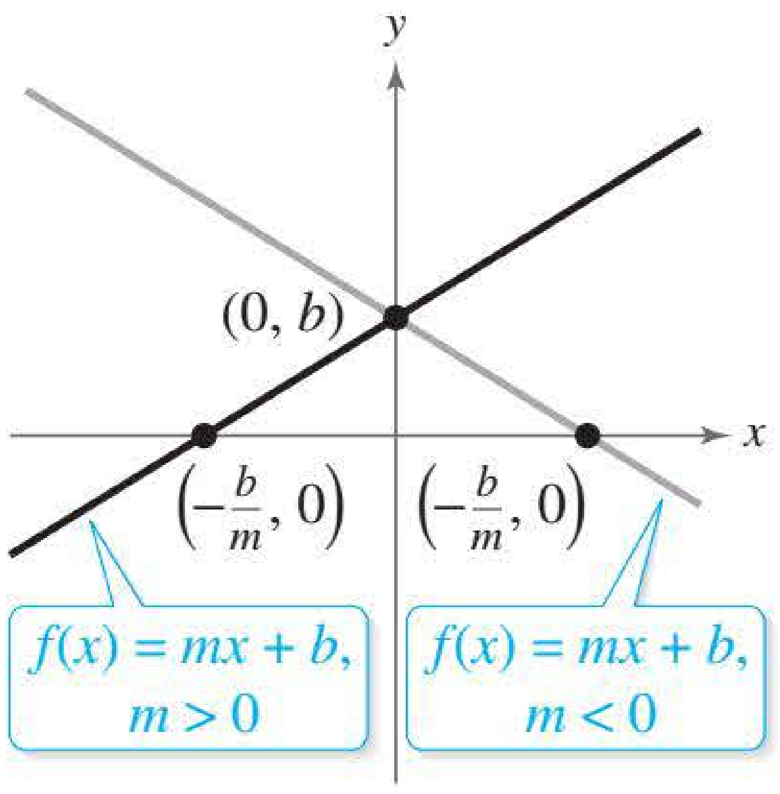
Ron Larson

GRAPHS OF PARENT FUNCTIONS

Linear Function

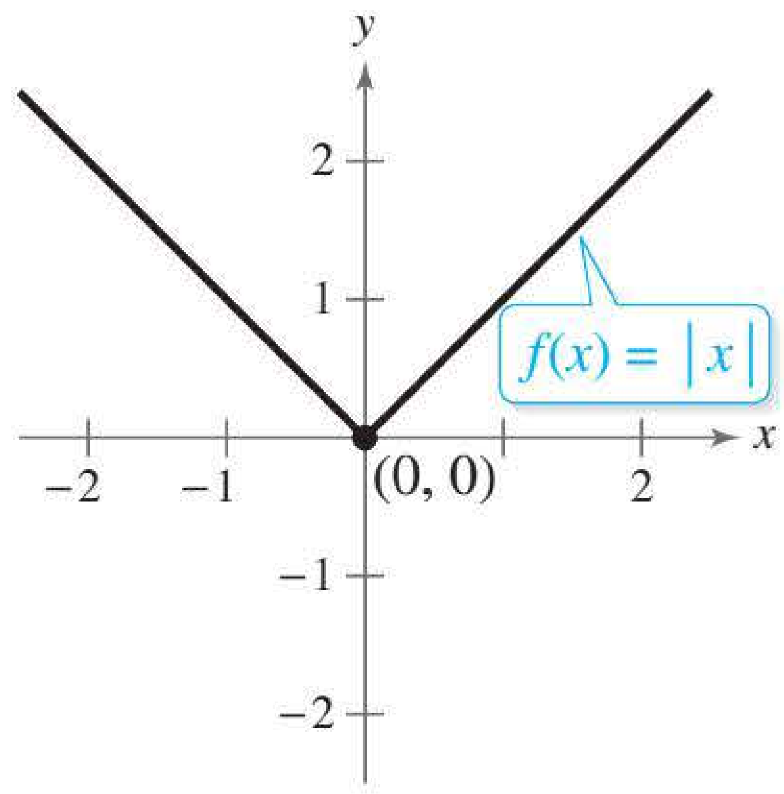
$f(x) = mx + b$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
x-intercept: $(-b/m, 0)$
y-intercept: $(0, b)$
Increasing when $m > 0$
Decreasing when $m < 0$

Absolute Value Function

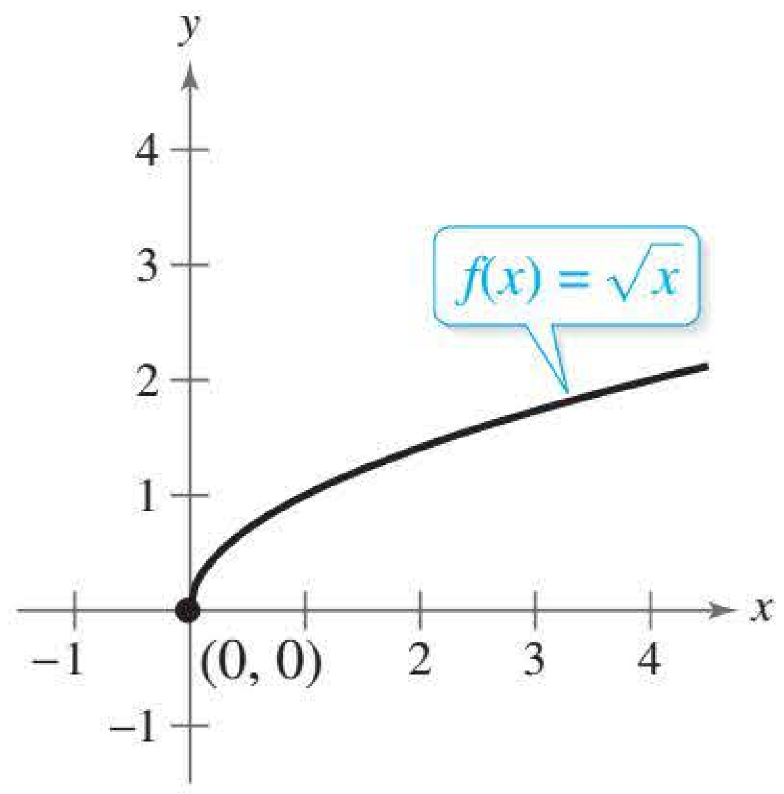
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



Domain: $(-\infty, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$
Increasing on $(0, \infty)$
Even function
y-axis symmetry

Square Root Function

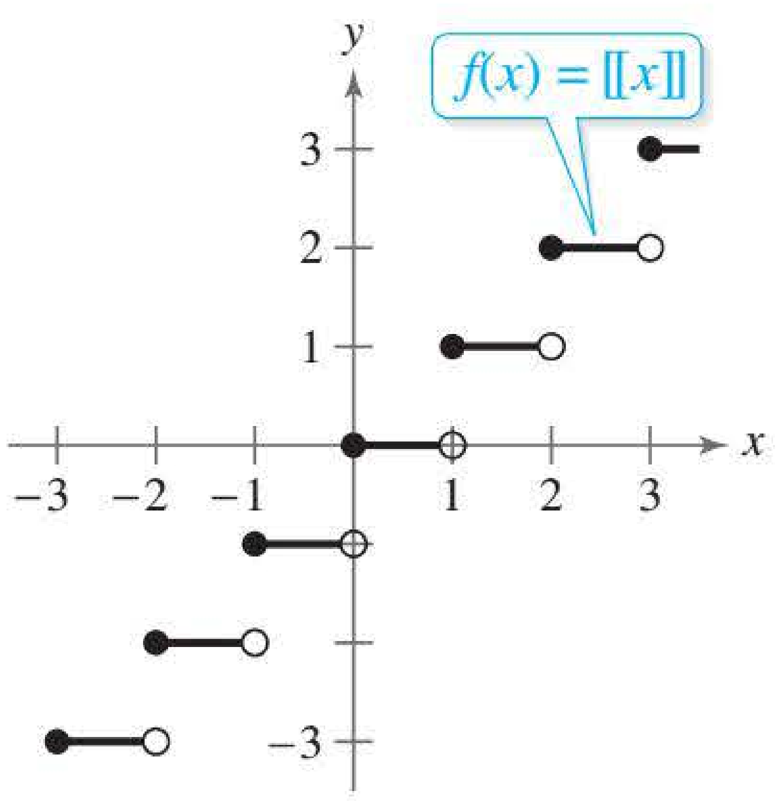
$f(x) = \sqrt{x}$



Domain: $[0, \infty)$
Range: $[0, \infty)$
Intercept: $(0, 0)$
Increasing on $(0, \infty)$

Greatest Integer Function

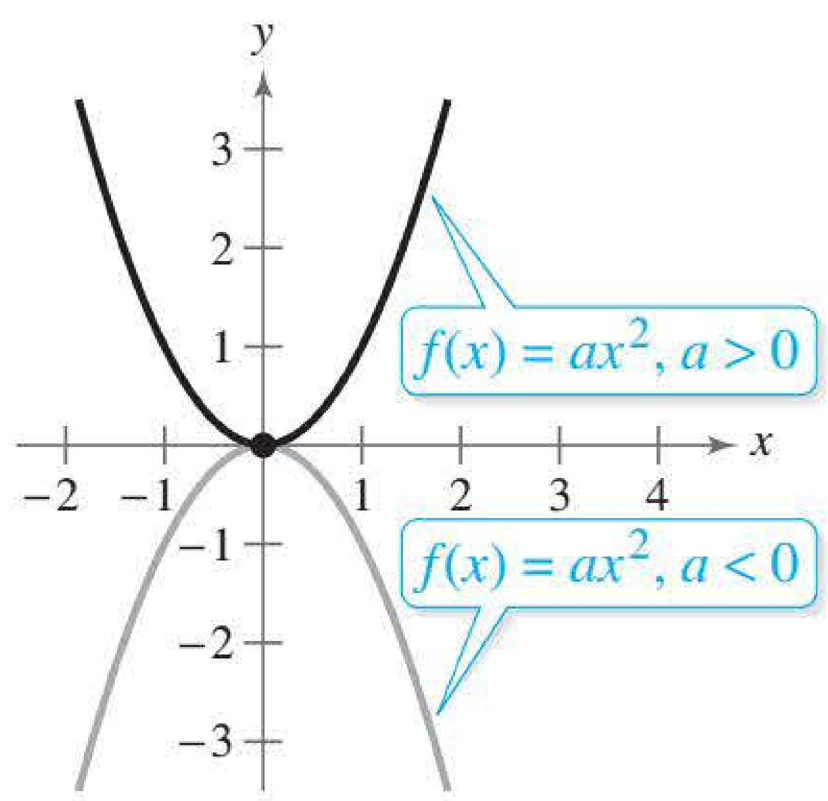
$f(x) = [\![x]\!]$



Domain: $(-\infty, \infty)$
Range: the set of integers
x-intercepts: in the interval $[0, 1)$
y-intercept: $(0, 0)$
Constant between each pair of consecutive integers
Jumps vertically one unit at each integer value

Quadratic (Squaring) Function

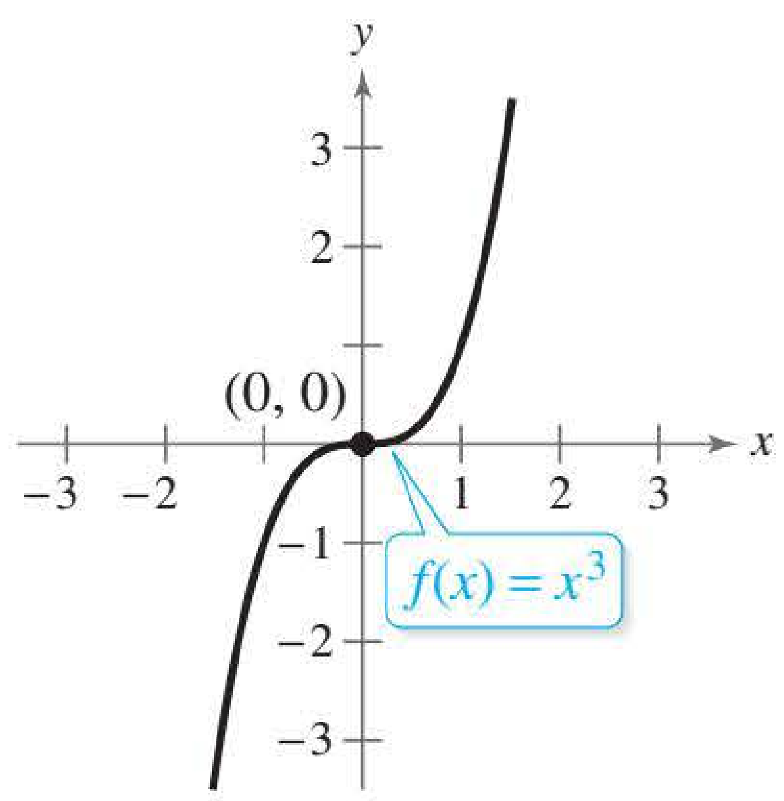
$f(x) = ax^2$



Domain: $(-\infty, \infty)$
Range $(a > 0)$: $[0, \infty)$
Range $(a < 0)$: $(-\infty, 0]$
Intercept: $(0, 0)$
Decreasing on $(-\infty, 0)$ for $a > 0$
Increasing on $(0, \infty)$ for $a > 0$
Increasing on $(-\infty, 0)$ for $a < 0$
Decreasing on $(0, \infty)$ for $a < 0$
Even function
y-axis symmetry
Relative minimum $(a > 0)$, relative maximum $(a < 0)$, or vertex: $(0, 0)$

Cubic Function

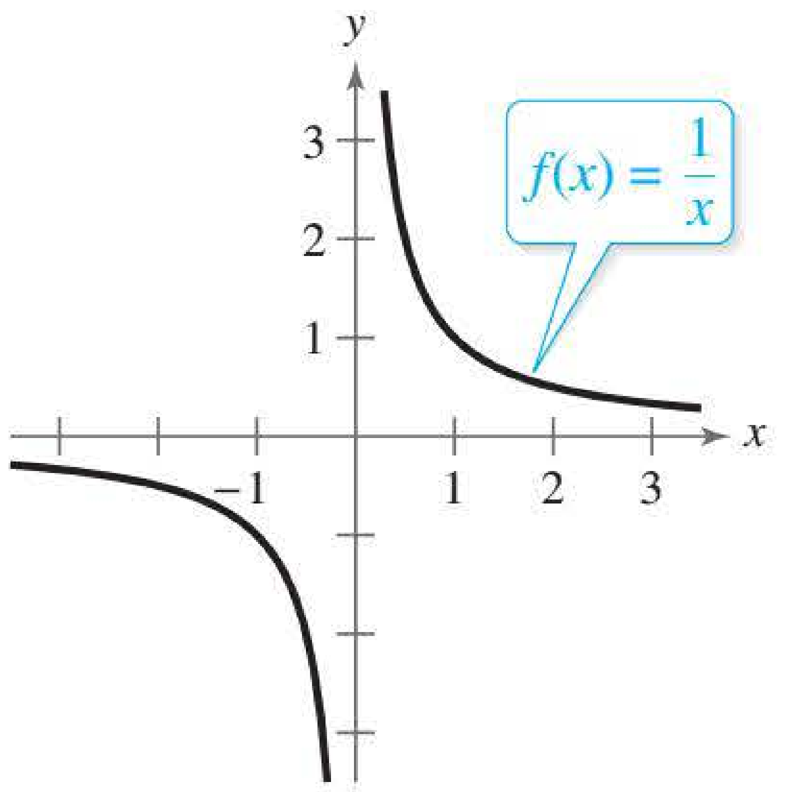
$f(x) = x^3$



Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(0, 0)$
Increasing on $(-\infty, \infty)$
Odd function
Origin symmetry

Rational (Reciprocal) Function

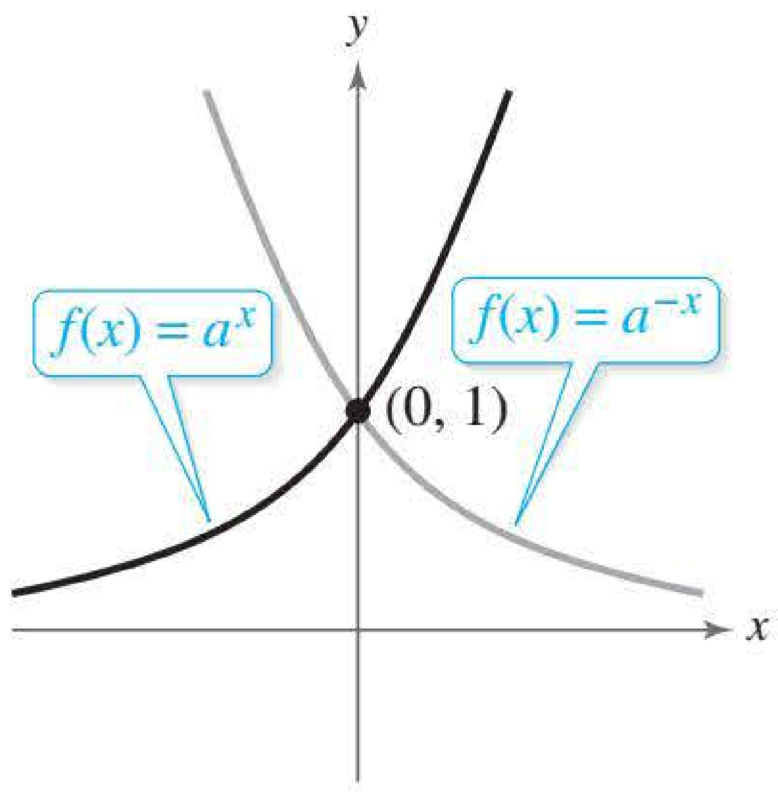
$f(x) = \dfrac{1}{x}$



Domain: $(-\infty, 0) \cup (0, \infty)$
Range: $(-\infty, 0) \cup (0, \infty)$
No intercepts
Decreasing on $(-\infty, 0)$ and $(0, \infty)$
Odd function
Origin symmetry
Vertical asymptote: y-axis
Horizontal asymptote: x-axis

Exponential Function

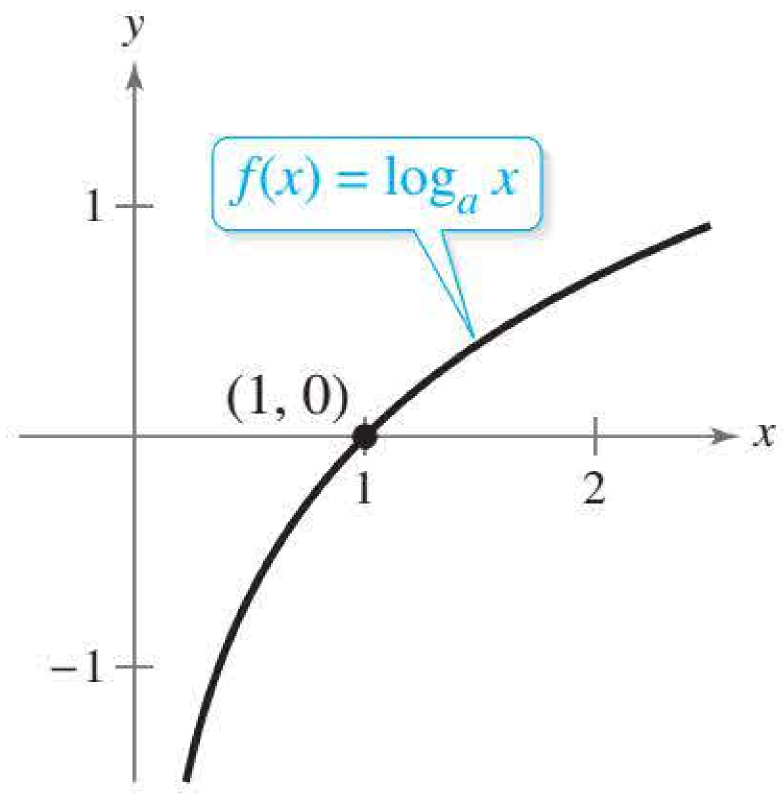
$f(x) = a^x,\ a > 1$



Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Intercept: $(0, 1)$
Increasing on $(-\infty, \infty)$
for $f(x) = a^x$
Decreasing on $(-\infty, \infty)$
for $f(x) = a^{-x}$
Horizontal asymptote: x-axis
Continuous

Logarithmic Function

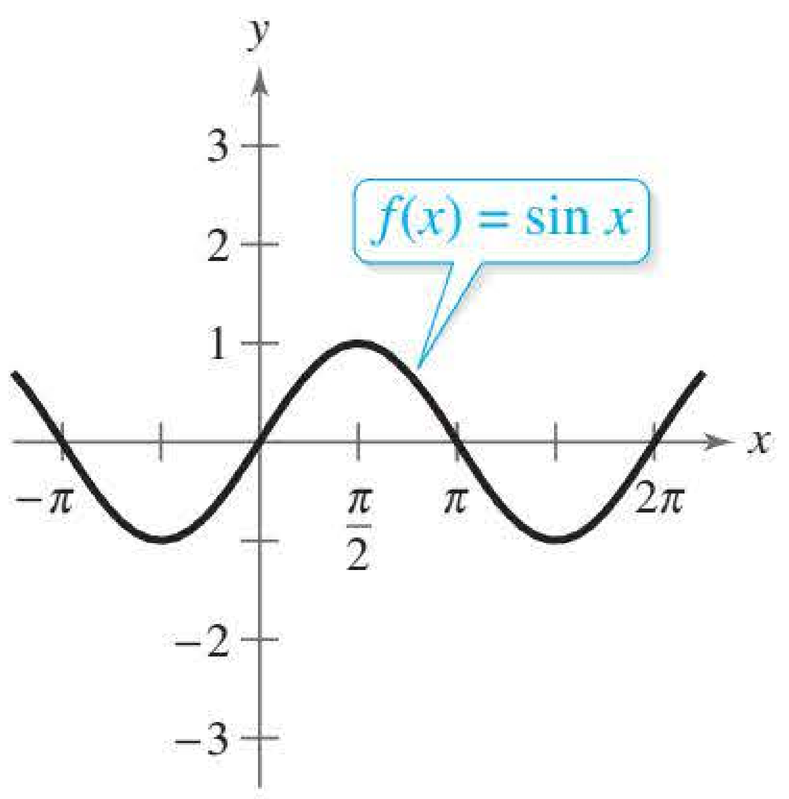
$f(x) = \log_a x,\ a > 1$



Domain: $(0, \infty)$
Range: $(-\infty, \infty)$
Intercept: $(1, 0)$
Increasing on $(0, \infty)$
Vertical asymptote: y-axis
Continuous
Reflection of graph of $f(x) = a^x$
in the line $y = x$

Sine Function

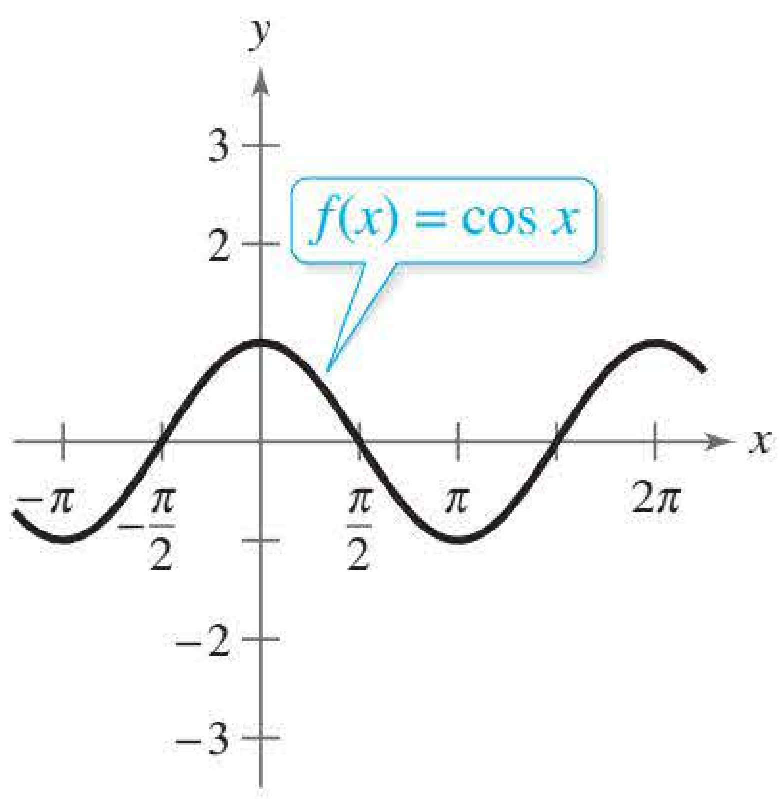
$f(x) = \sin x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Odd function
Origin symmetry

Cosine Function

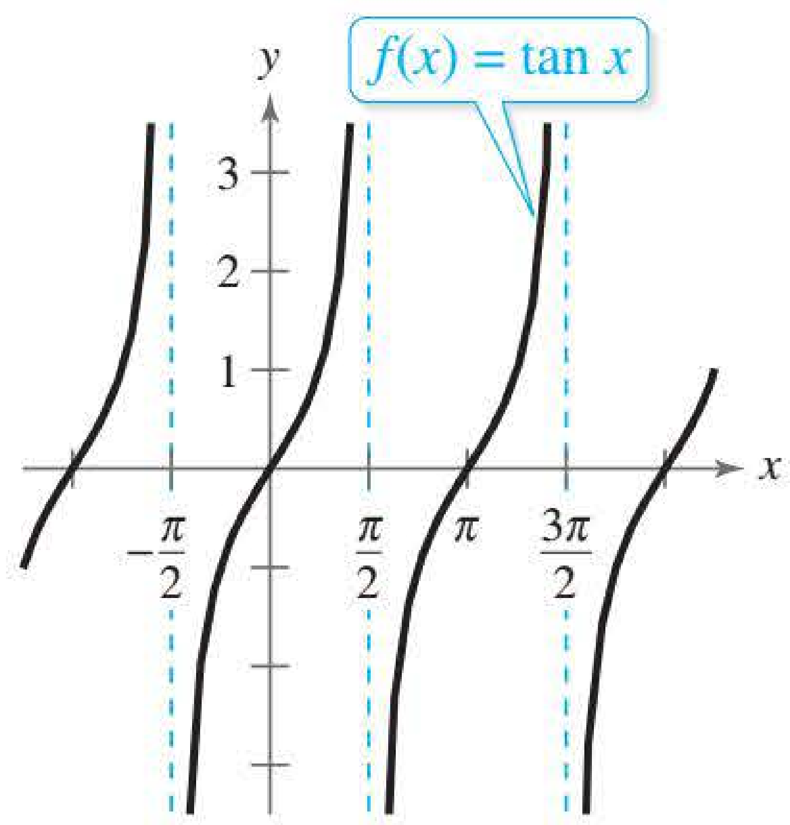
$f(x) = \cos x$



Domain: $(-\infty, \infty)$
Range: $[-1, 1]$
Period: 2π
x-intercepts: $\left(\dfrac{\pi}{2} + n\pi, 0\right)$
y-intercept: $(0, 1)$
Even function
y-axis symmetry

Tangent Function

$f(x) = \tan x$



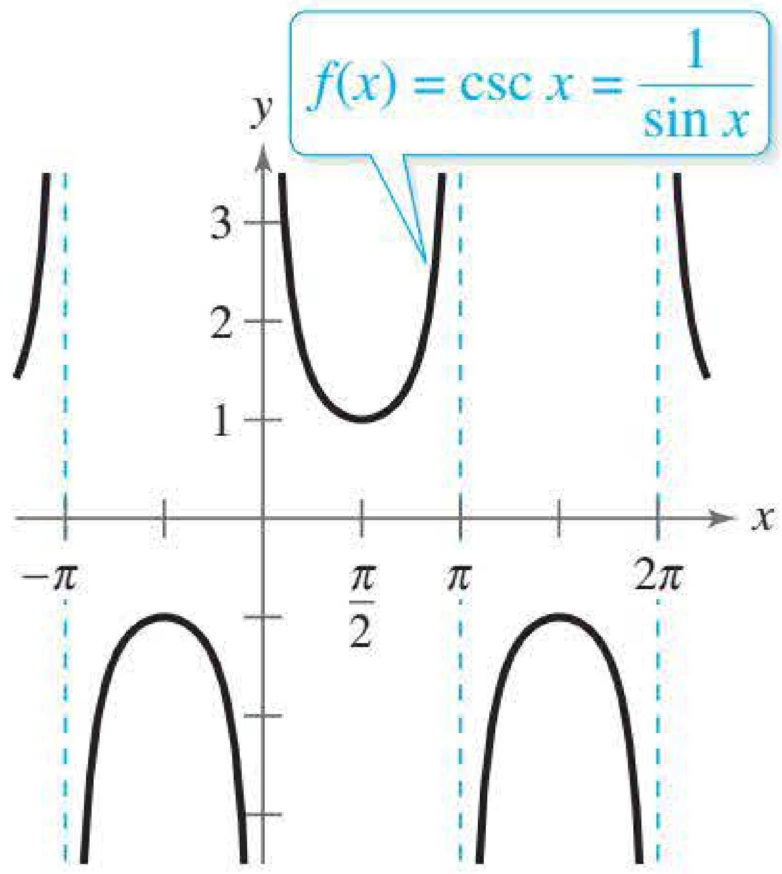
Domain: all $x \neq \dfrac{\pi}{2} + n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $(n\pi, 0)$
y-intercept: $(0, 0)$
Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Odd function
Origin symmetry

Cosecant Function

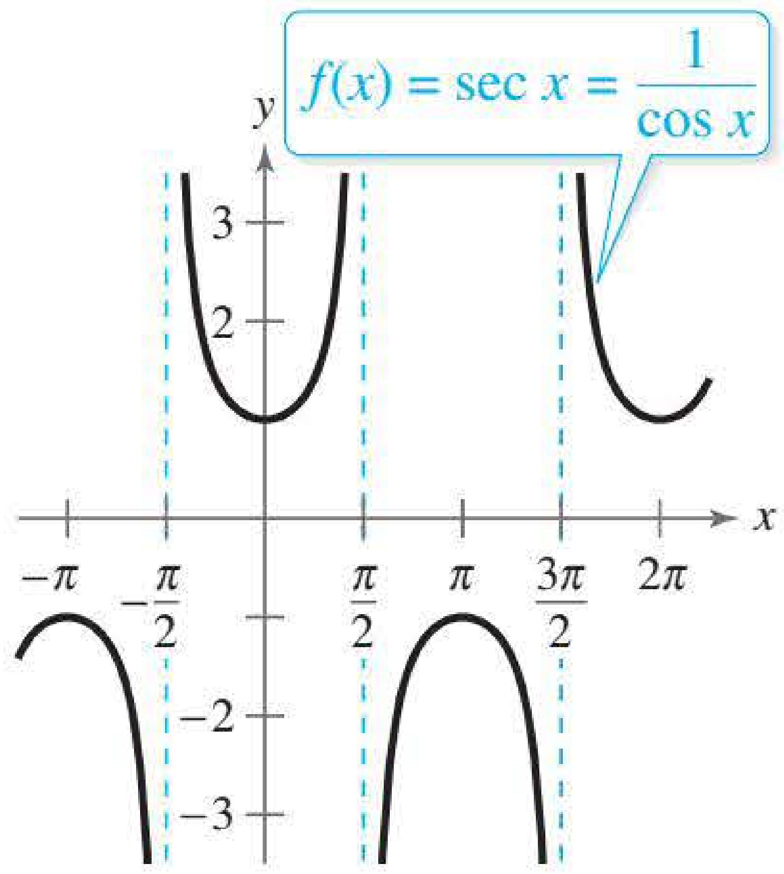
$f(x) = \csc x$



Domain: all $x \neq n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
No intercepts
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Secant Function

$f(x) = \sec x$



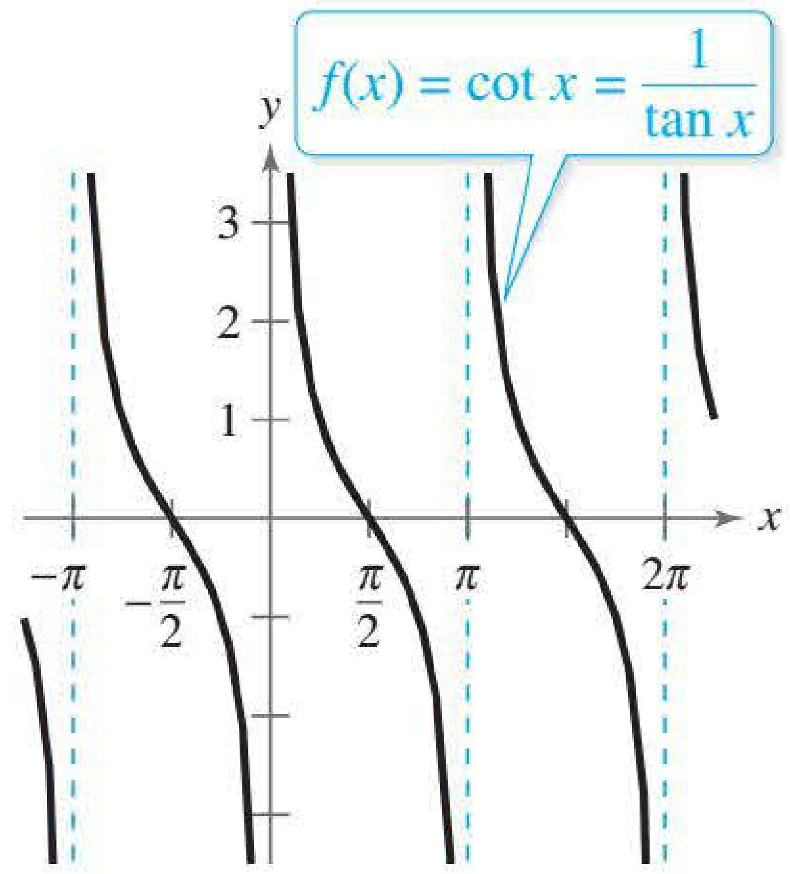
Domain: all $x \neq \frac{\pi}{2} + n\pi$
Range: $(-\infty, -1] \cup [1, \infty)$
Period: 2π
y-intercept: $(0, 1)$
Vertical asymptotes:

$$x = \frac{\pi}{2} + n\pi$$

Even function
y-axis symmetry

Cotangent Function

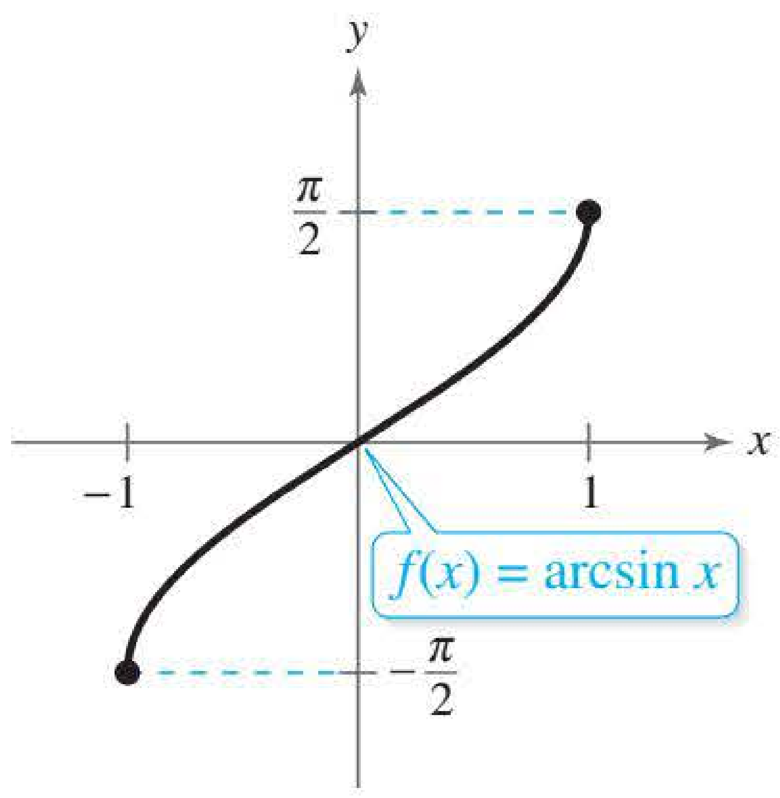
$f(x) = \cot x$



Domain: all $x \neq n\pi$
Range: $(-\infty, \infty)$
Period: π
x-intercepts: $\left(\frac{\pi}{2} + n\pi, 0\right)$
Vertical asymptotes: $x = n\pi$
Odd function
Origin symmetry

Inverse Sine Function

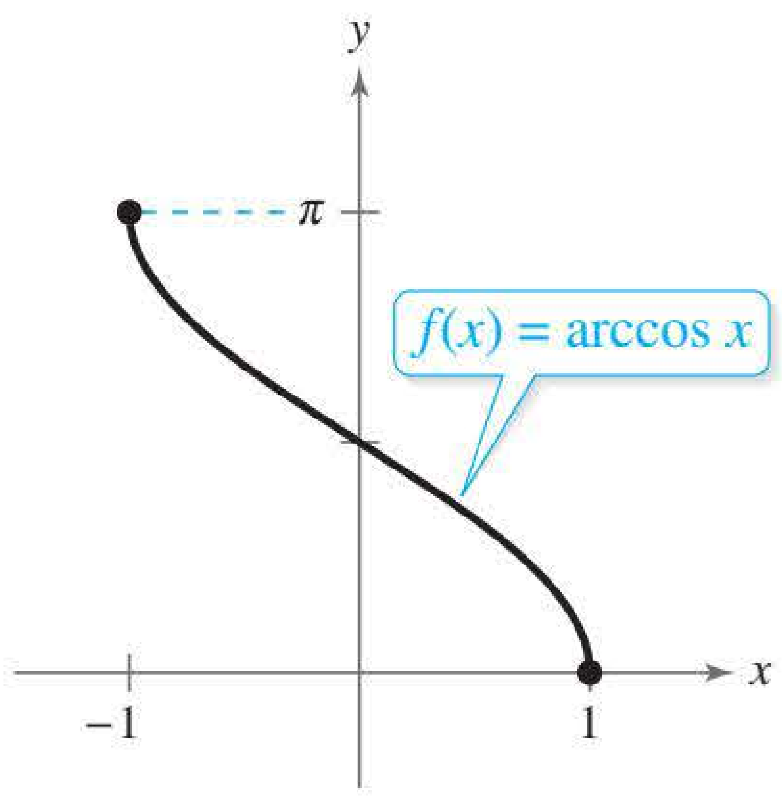
$f(x) = \arcsin x$



Domain: $[-1, 1]$
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
Intercept: $(0, 0)$
Odd function
Origin symmetry

Inverse Cosine Function

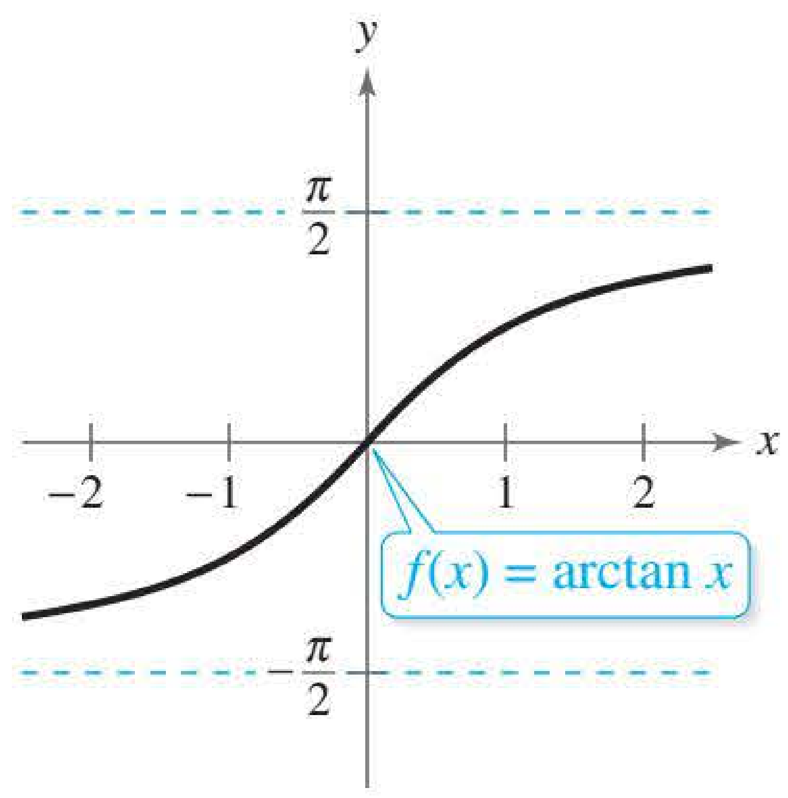
$f(x) = \arccos x$



Domain: $[-1, 1]$
Range: $[0, \pi]$
y-intercept: $\left(0, \frac{\pi}{2}\right)$

Inverse Tangent Function

$f(x) = \arctan x$



Domain: $(-\infty, \infty)$
Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Intercept: $(0, 0)$
Horizontal asymptotes:

$$y = \pm\frac{\pi}{2}$$

Odd function
Origin symmetry

Precalculus: A Concise Course

Third Edition

Precalculus: A Concise Course

Third Edition

Ron Larson
The Pennsylvania State University
The Behrend College

With the assistance of David C. Falvo
The Pennsylvania State University
The Behrend College

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Third Edition

Ron Larson

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Appendices

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Preface

Welcome to *Precalculus: A Concise Course*, Third Edition. I am proud to present to you this new edition. As with all editions, I have been able to incorporate many useful comments from you, our user. And while much has changed in this revision, you will still find what you expect—a pedagogically sound, mathematically precise, and comprehensive textbook. Additionally, I am pleased and excited to offer you something brand new—a companion website at **LarsonPrecalculus.com.**

My goal for every edition of this textbook is to provide students with the tools that they need to master precalculus. I hope you find that the changes in this edition, together with **LarsonPrecalculus.com,** will help accomplish just that.

New To This Edition

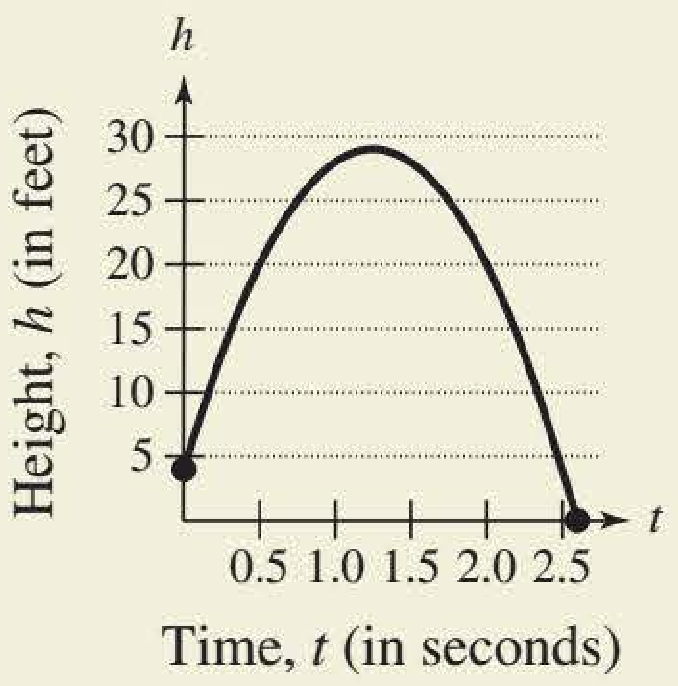
NEW LarsonPrecalculus.com

This companion website offers multiple tools and resources to supplement your learning. Access to these features is free. View and listen to worked-out solutions of Checkpoint problems in English or Spanish, download data sets, work on chapter projects, watch lesson videos, and much more.

NEW Chapter Opener

Each Chapter Opener highlights real-life applications used in the examples and exercises.

96. HOW DO YOU SEE IT? The graph represents the height h of a projectile after t seconds.



(a) Explain why h is a function of t.

(b) Approximate the height of the projectile after 0.5 second and after 1.25 seconds.

(c) Approximate the domain of h.

(d) Is t a function of h? Explain.

NEW Summarize

The Summarize feature at the end of each section helps you organize the lesson's key concepts into a concise summary, providing you with a valuable study tool.

NEW How Do You See It?

The How Do You See It? feature in each section presents a real-life exercise that you will solve by visual inspection using the concepts learned in the lesson. This exercise is excellent for classroom discussion or test preparation.

NEW Checkpoints

Accompanying every example, the Checkpoint problems encourage immediate practice and check your understanding of the concepts presented in the example. View and listen to worked-out solutions of the Checkpoint problems in English or Spanish at LarsonPrecalculus.com.

NEW Data Spreadsheets

Download these editable spreadsheets from LarsonPrecalculus.com, and use the data to solve exercises.

DATA

Year	Number of Tax Returns Made Through E-File
2003	52.9
2004	61.5
2005	68.5
2006	73.3
2007	80.0
2008	89.9
2009	95.0
2010	98.7

Spreadsheet at LarsonPrecalculus.com

REVISED Exercise Sets

The exercise sets have been carefully and extensively examined to ensure they are rigorous and relevant and to include all topics our users have suggested. The exercises have been **reorganized and titled** so you can better see the connections between examples and exercises. Multi-step, real-life exercises reinforce problem-solving skills and mastery of concepts by giving you the opportunity to apply the concepts in real-life situations.

REVISED Section Objectives

A bulleted list of learning objectives provides you the opportunity to preview what will be presented in the upcoming section.

REVISED Remark

These hints and tips reinforce or expand upon concepts, help you learn how to study mathematics, caution you about common errors, address special cases, or show alternative or additional steps to a solution of an example.

Calc Chat

For the past several years, an independent website—CalcChat.com—has provided free solutions to all odd-numbered problems in the text. Thousands of students have visited the site for practice and help with their homework. For this edition, I used information from CalcChat.com, including which solutions students accessed most often, to help guide the revision of the exercises.

Trusted Features

Side-By-Side Examples

Throughout the text, we present solutions to many examples from multiple perspectives—algebraically, graphically, and numerically. The side-by-side format of this pedagogical feature helps you to see that a problem can be solved in more than one way and to see that different methods yield the same result. The side-by-side format also addresses many different learning styles.

Algebra Help

Algebra Help directs you to sections of the textbook where you can review algebra skills needed to master the current topic.

Technology

The technology feature gives suggestions for effectively using tools such as calculators, graphing calculators, and spreadsheet programs to help deepen your understanding of concepts, ease lengthy calculations, and provide alternate solution methods for verifying answers obtained by hand.

Historical Notes

These notes provide helpful information regarding famous mathematicians and their work.

Algebra of Calculus

Throughout the text, special emphasis is given to the algebraic techniques used in calculus. Algebra of Calculus examples and exercises are integrated throughout the text and are identified by the symbol f.

Vocabulary Exercises

The vocabulary exercises appear at the beginning of the exercise set for each section. These problems help you review previously learned vocabulary terms that you will use in solving the section exercises.

Project: Department of Defense The table shows the total numbers of military personnel P (in thousands) on active duty from 1980 through 2010. *(Source: U.S. Department of Defense)*

Year	Personnel, P	Year	Personnel, P
1980	2051	1995	1518
1981	2083	1996	1472
1982	2109	1997	1439
1983	2123	1998	1407
1984	2138	1999	1386
1985	2151	2000	1384
1986	2169	2001	1385
1987	2174	2002	1414
1988	2138	2003	1434
1989	2130	2004	1427
1990	2044	2005	1389
1991	1986	2006	1385
1992	1807	2007	1380
1993	1705	2008	1402
1994	1610	2009	1419
		2010	1431

(a) Use a graphing utility to plot the data. Let t represent the year, with $t = 0$ corresponding to 1980.

(b) A model that approximates the data is given by

$$P = \frac{9.6518t^2 - 244.743t + 2044.77}{0.0059t^2 - 0.131t + 1}$$

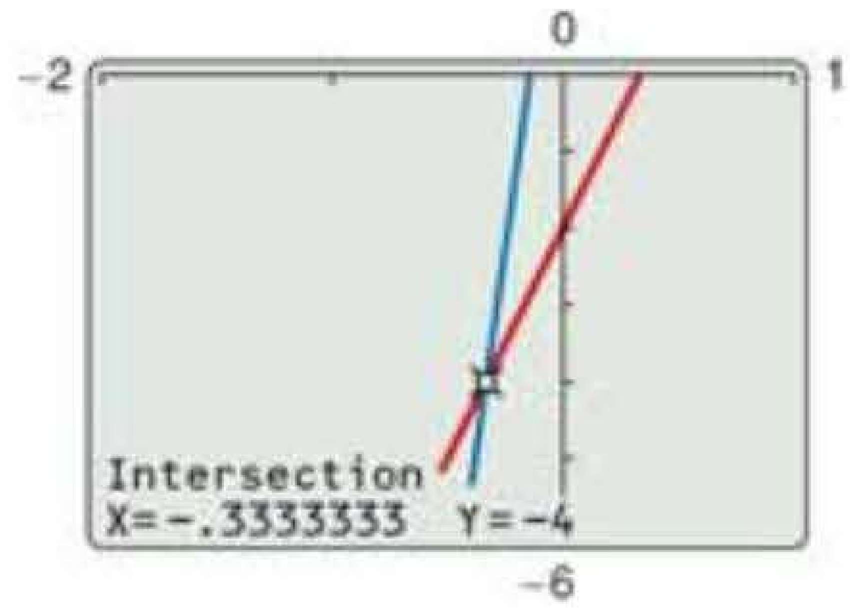
where P is the total number of personnel (in thousands) and t is the year, with $t = 0$ corresponding to 1980. Construct a table showing the actual values of P and the values of P obtained using the model.

TECHNOLOGY You can use a graphing utility to check that a solution is reasonable. One way to do this is to graph the left side of the equation, then graph the right side of the equation, and determine the point of intersection. For instance, in Example 2, if you graph the equations

$y_1 = 6(x - 1) + 4$ The left side

$y_2 = 3(7x + 1)$ The right side

in the same viewing window, they should intersect at $x = -\frac{1}{3}$, as shown in the graph below.



Project

The projects at the end of selected sections involve in-depth applied exercises in which you will work with large, real-life data sets, often creating or analyzing models. These projects are offered online at LarsonPrecalculus.com.

Chapter Summaries

The Chapter Summary now includes explanations and examples of the objectives taught in each chapter.

Enhanced WebAssign

Enhanced WebAssign combines exceptional Precalculus content that you know and love with the most powerful online homework solution, WebAssign. Enhanced WebAssign engages you with immediate feedback, rich tutorial content and interactive, fully customizable eBooks (YouBook) helping you to develop a deeper conceptual understanding of the subject matter.

Instructor Resources

Print

Annotated Instructor's Edition
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This AIE is the complete student text plus point-of-use annotations for you, including extra projects, classroom activities, teaching strategies, and additional examples. Answers to even-numbered text exercises, Vocabulary Checks, and Explorations are also provided.

Complete Solutions Manual
ISBN-13: 978-1-133-95450-7

This manual contains solutions to all exercises from the text, including Chapter Review Exercises, and Chapter Tests.

Media

PowerLecture with ExamView™
ISBN-13: 978-1-133-95998-4

The DVD provides you with dynamic media tools for teaching Precalculus: A Concise Course while using an interactive white board. PowerPoint® lecture slides and art slides of the figures from the text, together with electronic files for the test bank and a link to the Solution Builder, are available. The algorithmic ExamView allows you to create, deliver, and customize tests (both print and online) in minutes with this easy-to-use assessment system. The DVD also provides you with a tutorial on integrating our instructor materials into your interactive whiteboard platform. Enhance how your students interact with you, your lecture, and each other.

Solution Builder
(*www.cengage.com/solutionbuilder*)
This online instructor database offers complete worked-out solutions to all exercises in the text, allowing you to create customized, secure solutions printouts (in PDF format) matched exactly to the problems you assign in class.

ENHANCED WebAssign

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Student Resources

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Student Study and Solutions Manual
ISBN-13: 978-1-133-95449-1

This guide offers step-by-step solutions for all odd-numbered text exercises, Chapter and Cumulative Tests, and Practice Tests with solutions.

Text-Specific DVD
ISBN-13: 978-1-285-17767-0

Keyed to the text by section, these DVDs provide comprehensive coverage of the course—along with additional explanations of concepts, sample problems, and application—to help you review essential topics.

Media

www.webassign.net
Printed Access Card: 978-0-538-73810-1
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Enhanced WebAssign (assigned by the instructor) provides you with instant feedback on homework assignments. This online homework system is easy to use and includes helpful links to textbook sections, video examples, and problem-specific tutorials.

CengageBrain.com

Visit *www.cengagebrain.com* to access additional course materials and companion resources. At the CengageBrain.com home page, search for the ISBN of your title (from the back cover of your book) using the search box at the top of the page. This will take you to the product page where free companion resources can be found.

Acknowledgements

I would like to thank the many people who have helped me prepare the text and the supplements package. Their encouragement, criticisms, and suggestions have been invaluable.

Thank you to all of the instructors who took the time to review the changes in this edition and to provide suggestions for improving it. Without your help, this book would not be possible.

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On a personal level, I am grateful to my spouse, Deanna Gilbert Larson, for her love, patience, and support. Also, a special thanks goes to R. Scott O'Neil. If you have suggestions for improving this text, please feel free to write to me. Over the past two decades I have received many useful comments from both instructors and students, and I value these comments very highly.

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Penn State University
www.RonLarson.com

1 Functions and Their Graphs

Bacteria *(Example 8, page 80)*

Snowstorm *(Exercise 47, page 66)*

Average Speed *(Example 7, page 54)*

Alternative-Fueled Vehicles *(Example 10, page 42)*

Americans with Disabilities Act *(page 28)*

1.1 Rectangular Coordinates

The Cartesian plane can help you visualize relationships between two variables. For instance, in Exercise 37 on page 9, given how far north and west one city is from another, plotting points to represent the cities can help you visualize these distances and determine the flying distance between the cities.

- **Plot points in the Cartesian plane.**
- **Use the Distance Formula to find the distance between two points.**
- **Use the Midpoint Formula to find the midpoint of a line segment.**
- **Use a coordinate plane to model and solve real-life problems.**

The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** named after the French mathematician René Descartes (1596–1650).

Two real number lines intersecting at right angles form the Cartesian plane, as shown in Figure 1.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**

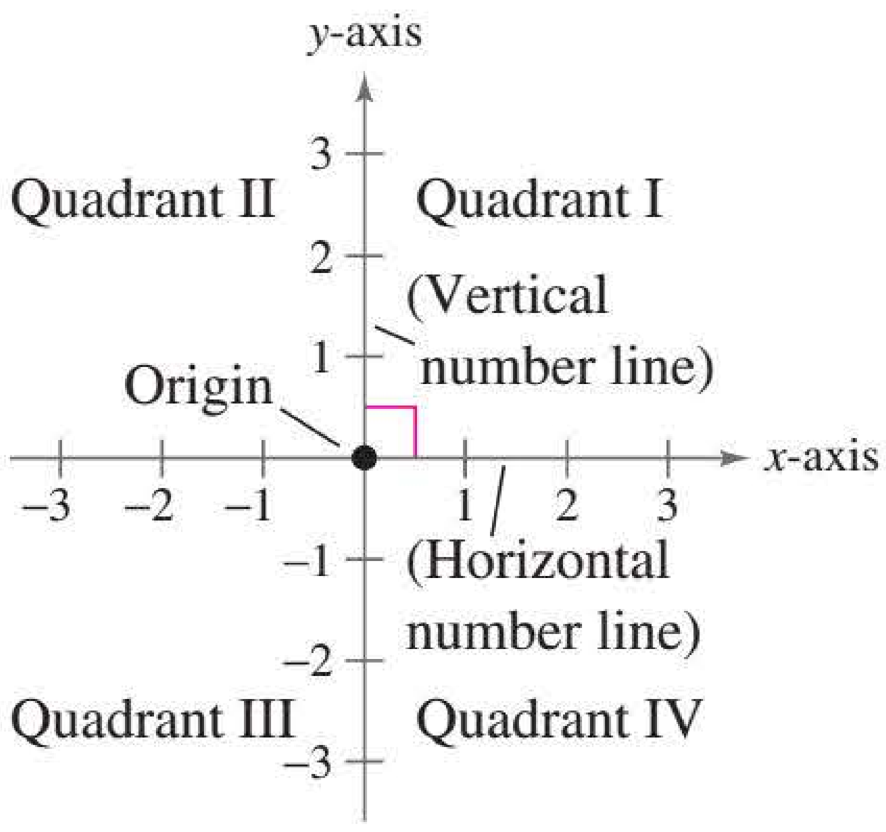


Figure 1.1

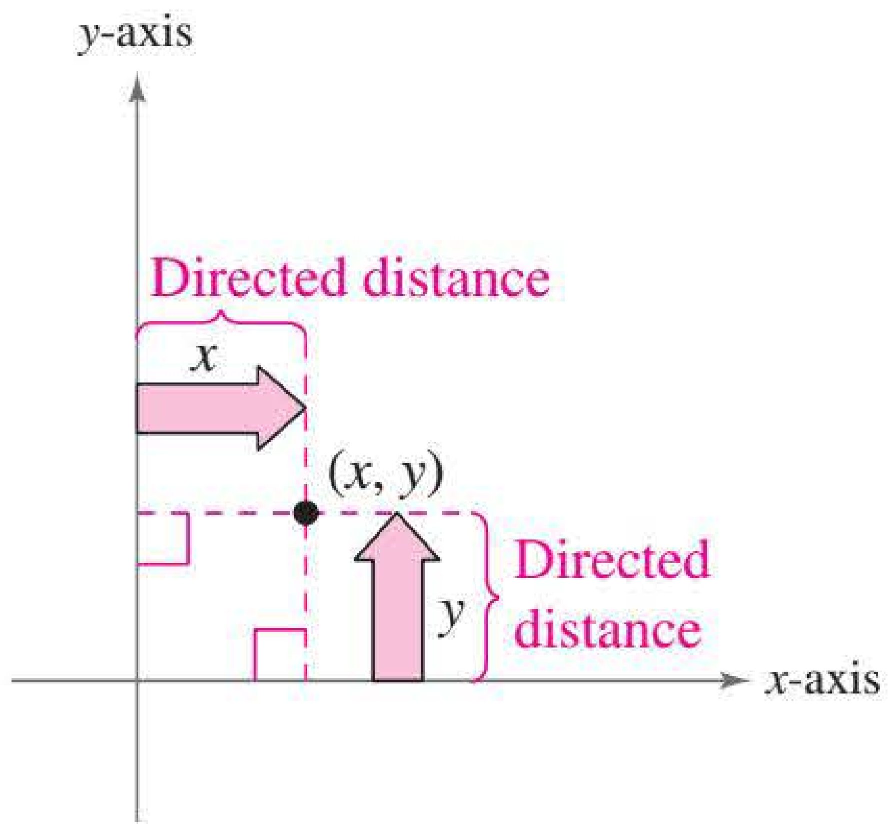
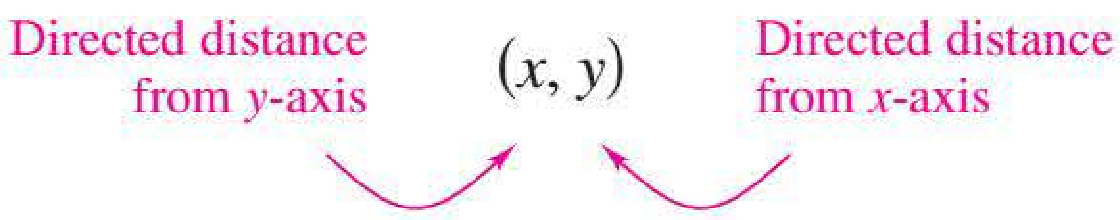


Figure 1.2

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure 1.2.



The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

EXAMPLE 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

Solution To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. Plot the other four points in a similar way, as shown in Figure 1.3.

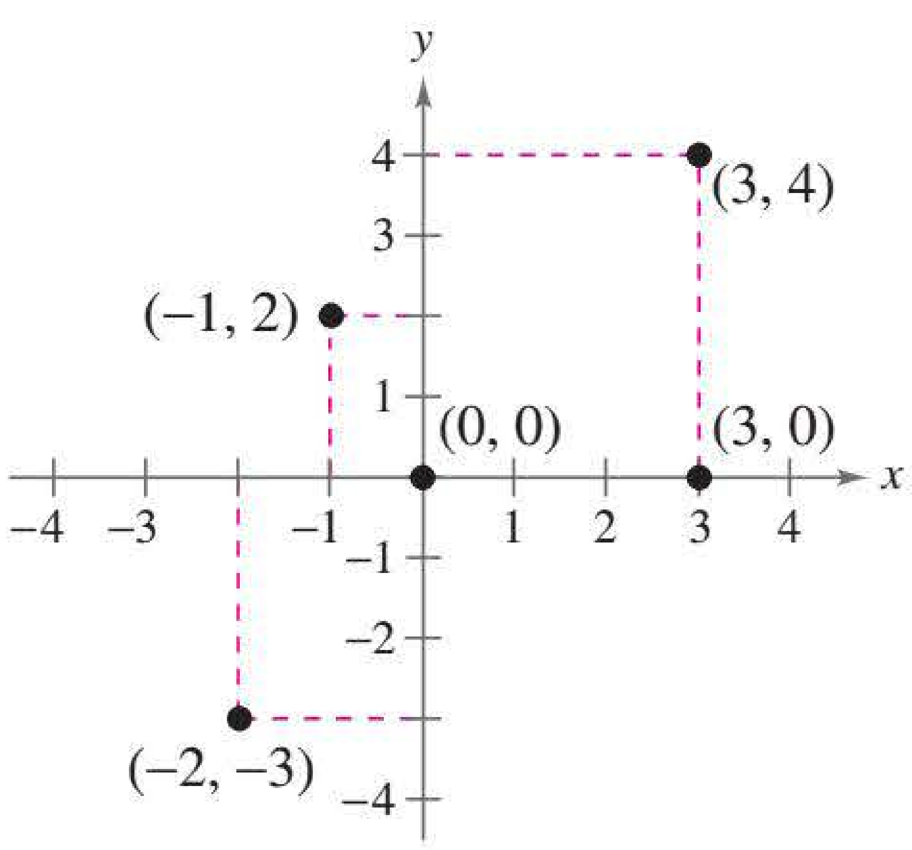


Figure 1.3

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Plot the points $(-3, 2)$, $(4, -2)$, $(3, 1)$, $(0, -2)$, and $(-1, -2)$.

Fernando Jose Vasconcelos Soares/Shutterstock.com

The beauty of a rectangular coordinate system is that it allows you to *see* relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates in the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

EXAMPLE 2 Sketching a Scatter Plot

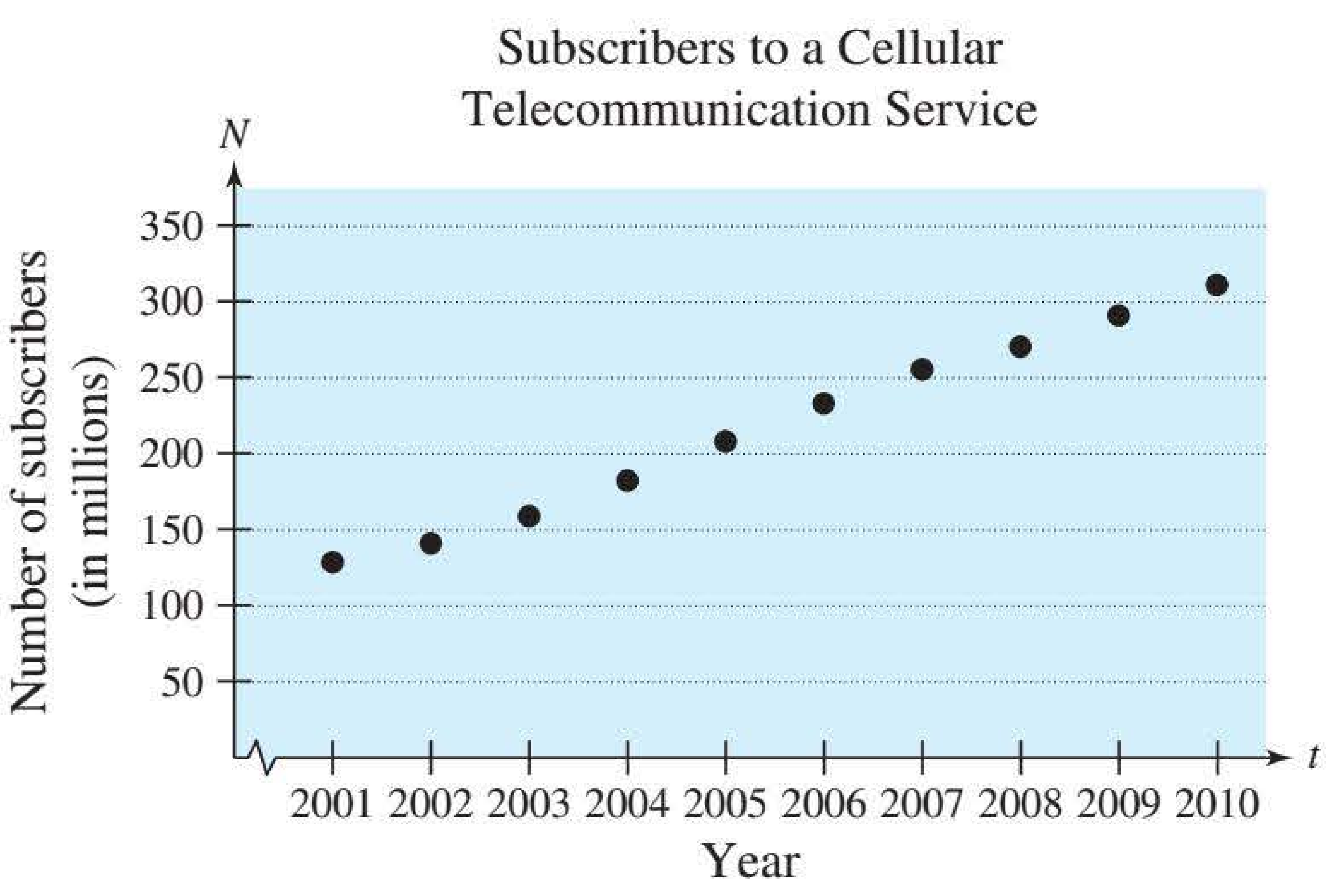
DATA

Year, t	Subscribers, N
2001	128.4
2002	140.8
2003	158.7
2004	182.1
2005	207.9
2006	233.0
2007	255.4
2008	270.3
2009	290.9
2010	311.0

Spreadsheet at LarsonPrecalculus.com

The table shows the numbers N (in millions) of subscribers to a cellular telecommunication service in the United States from 2001 through 2010, where t represents the year. Sketch a scatter plot of the data. *(Source: CTIA-The Wireless Association)*

Solution To sketch a *scatter plot* of the data shown in the table, represent each pair of values by an ordered pair (t, N) and plot the resulting points, as shown below. For instance, the ordered pair (2001, 128.4) represents the first pair of values. Note that the break in the t-axis indicates omission of the years before 2001.



Checkpoint *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

The table shows the numbers N (in thousands) of cellular telecommunication service employees in the United States from 2001 through 2010, where t represents the year. Sketch a scatter plot of the data. *(Source: CTIA-The Wireless Association)*

DATA

t	N
2001	203.6
2002	192.4
2003	205.6
2004	226.0
2005	233.1
2006	253.8
2007	266.8
2008	268.5
2009	249.2
2010	250.4

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TECHNOLOGY The scatter plot in Example 2 is only one way to represent the data graphically. You could also represent the data using a bar graph or a line graph. Try using a graphing utility to represent the data given in Example 2 graphically.

In Example 2, you could have let $t = 1$ represent the year 2001. In that case, there would not have been a break in the horizontal axis, and the labels 1 through 10 (instead of 2001 through 2010) would have been on the tick marks.

The Pythagorean Theorem and the Distance Formula

The following famous theorem is used extensively throughout this course.

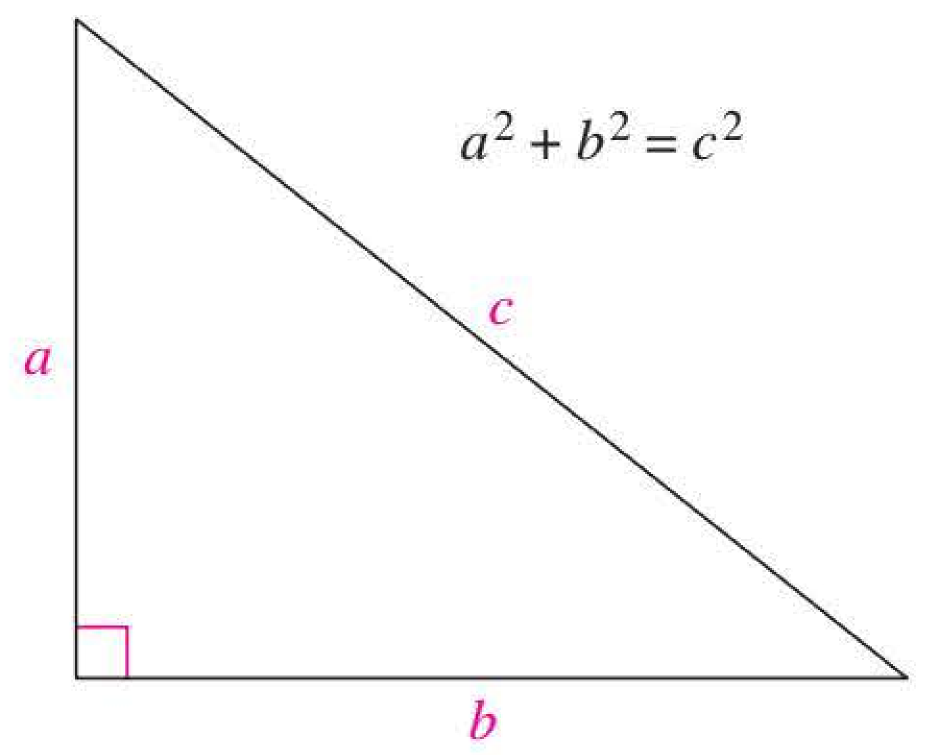


Figure 1.4

> **Pythagorean Theorem**
>
> For a right triangle with hypotenuse of length c and sides of lengths a and b, you have $a^2 + b^2 = c^2$, as shown in Figure 1.4. (The converse is also true. That is, if $a^2 + b^2 = c^2$, then the triangle is a right triangle.)

Suppose you want to determine the distance d between two points (x_1, y_1) and (x_2, y_2) in the plane. These two points can form a right triangle, as shown in Figure 1.5. The length of the vertical side of the triangle is $|y_2 - y_1|$ and the length of the horizontal side is $|x_2 - x_1|$.

By the Pythagorean Theorem,

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is the **Distance Formula.**

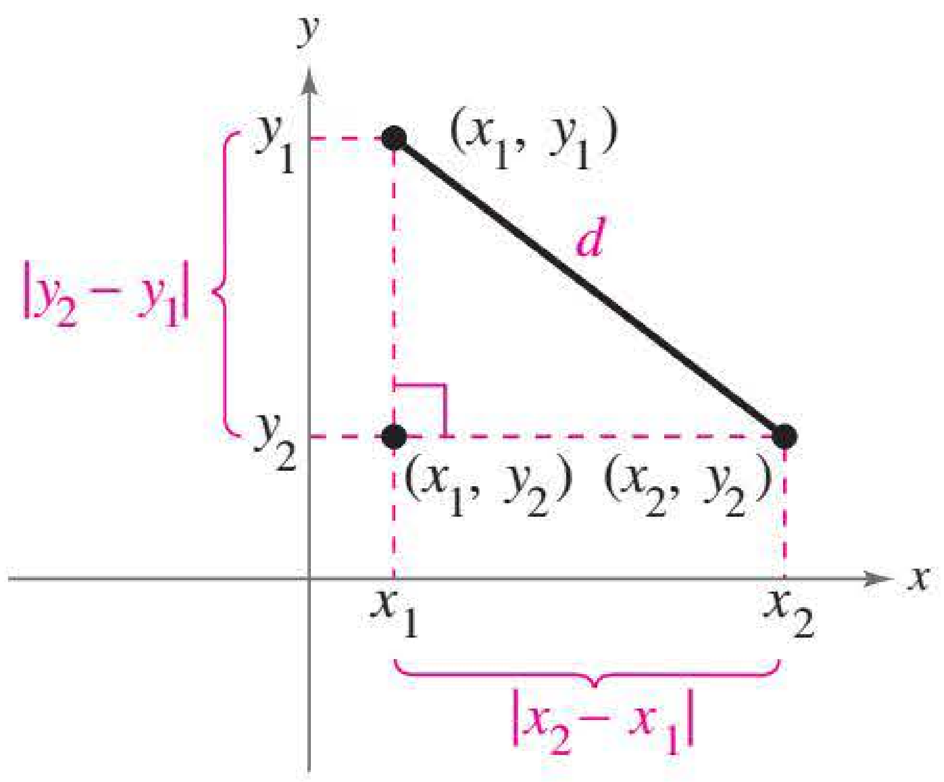


Figure 1.5

> **The Distance Formula**
>
> The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is
>
> $$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 3 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

Algebraic Solution

Let

$$(x_1, y_1) = (-2, 1) \quad \text{and} \quad (x_2, y_2) = (3, 4).$$

Then apply the Distance Formula.

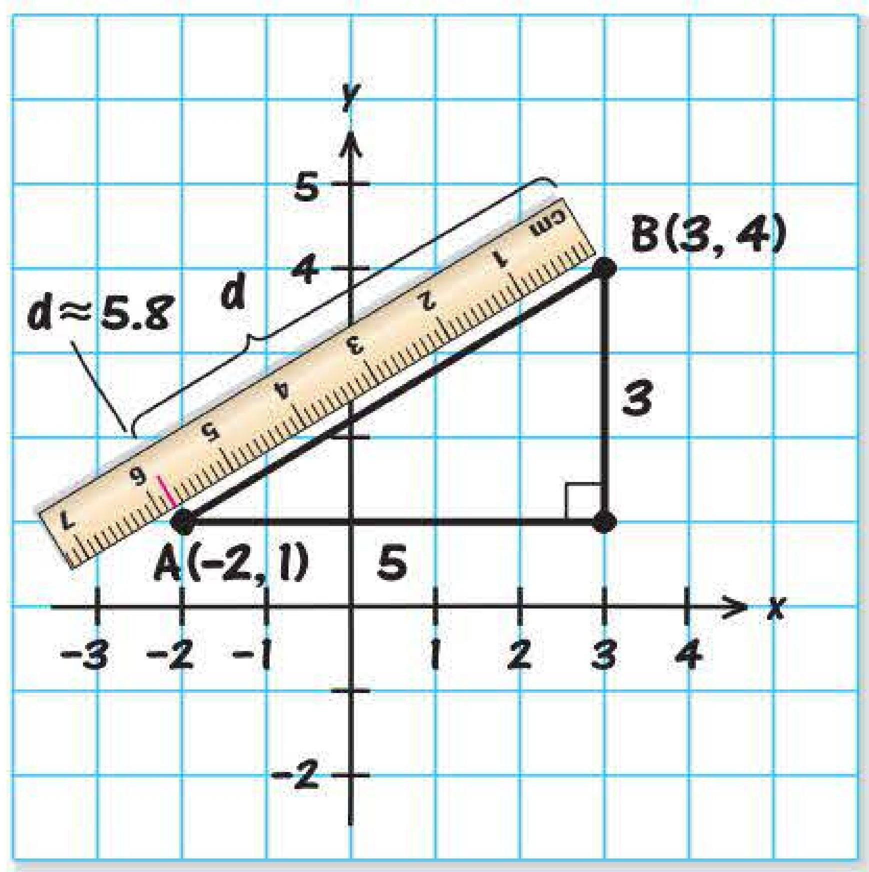
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	Distance Formula
$= \sqrt{[3 - (-2)]^2 + (4 - 1)^2}$	Substitute for x_1, y_1, x_2, and y_2.
$= \sqrt{(5)^2 + (3)^2}$	Simplify.
$= \sqrt{34}$	Simplify.
≈ 5.83	Use a calculator.

So, the distance between the points is about 5.83 units. Use the Pythagorean Theorem to check that the distance is correct.

$d^2 \stackrel{?}{=} 5^2 + 3^2$	Pythagorean Theorem
$(\sqrt{34})^2 \stackrel{?}{=} 5^2 + 3^2$	Substitute for d.
$34 = 34$	Distance checks. ✓

Graphical Solution

Use centimeter graph paper to plot the points $A(-2, 1)$ and $B(3, 4)$. Carefully sketch the line segment from A to B. Then use a centimeter ruler to measure the length of the segment.



The line segment measures about 5.8 centimeters. So, the distance between the points is about 5.8 units.

✓ **Checkpoint** Audio-video solution in English & Spanish at LarsonPrecalculus.com.

Find the distance between the points $(3, 1)$ and $(-3, 0)$.

EXAMPLE 4 Verifying a Right Triangle

Show that the points

(2, 1), (4, 0), and (5, 7)

are vertices of a right triangle.

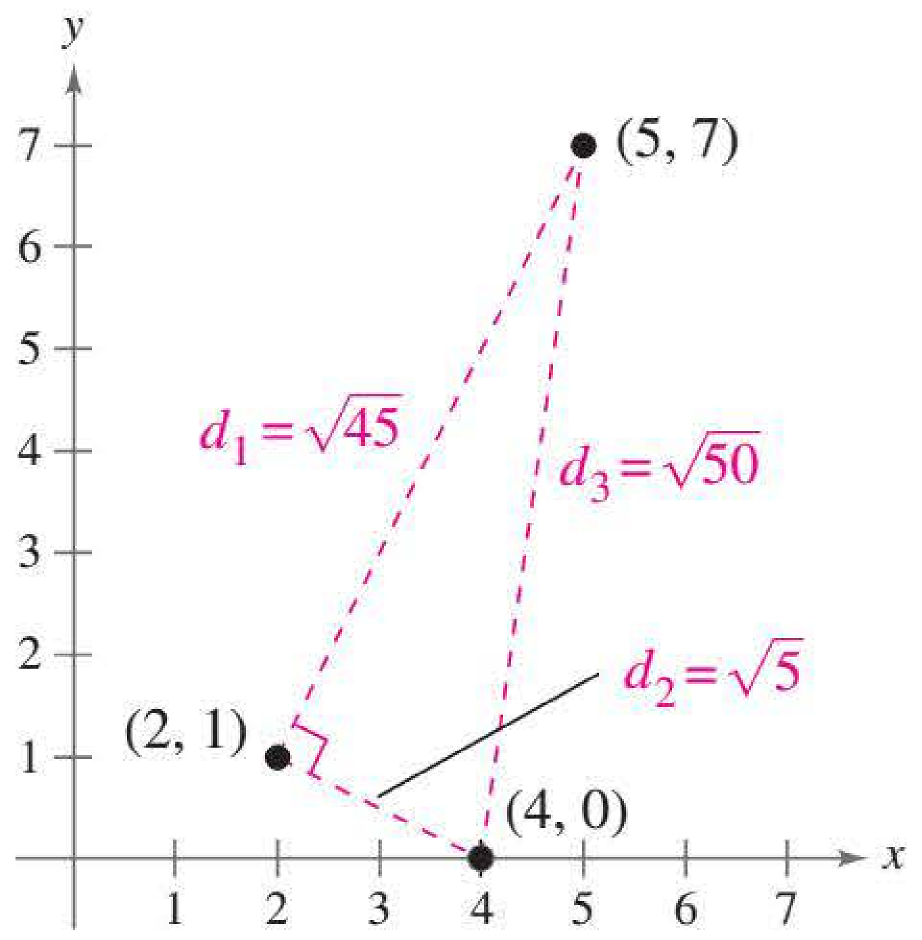


Figure 1.6

Solution The three points are plotted in Figure 1.6. Using the Distance Formula, the lengths of the three sides are as follows.

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9+36} = \sqrt{45}$$
$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4+1} = \sqrt{5}$$
$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1+49} = \sqrt{50}$$

Because $(d_1)^2 + (d_2)^2 = 45 + 5 = 50 = (d_3)^2$, you can conclude by the Pythagorean Theorem that the triangle must be a right triangle.

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Show that the points $(2, -1)$, $(5, 5)$, and $(6, -3)$ are vertices of a right triangle.

▷ **ALGEBRA HELP** You can review the techniques for evaluating a radical in Appendix A.2.

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, you can find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.**

The Midpoint Formula

The midpoint of the line segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

For a proof of the Midpoint Formula, see Proofs in Mathematics on page 110.

EXAMPLE 5 Finding a Line Segment's Midpoint

Find the midpoint of the line segment joining the points

$(-5, -3)$ and $(9, 3)$.

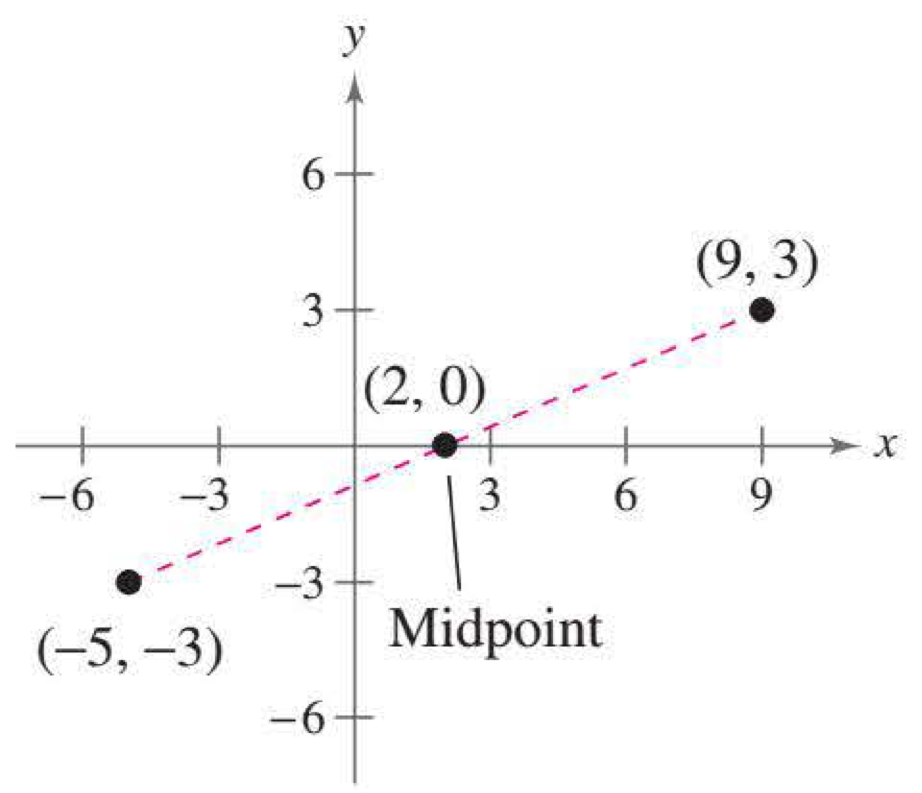

Figure 1.7

Solution Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \qquad \text{Midpoint Formula}$$
$$= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) \qquad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$
$$= (2, 0) \qquad \text{Simplify.}$$

The midpoint of the line segment is (2, 0), as shown in Figure 1.7.

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Find the midpoint of the line segment joining the points $(-2, 8)$ and $(4, -10)$.

Applications

EXAMPLE 6 Finding the Length of a Pass

A football quarterback throws a pass from the 28-yard line, 40 yards from the sideline. A wide receiver catches the pass on the 5-yard line, 20 yards from the same sideline, as shown in Figure 1.8. How long is the pass?

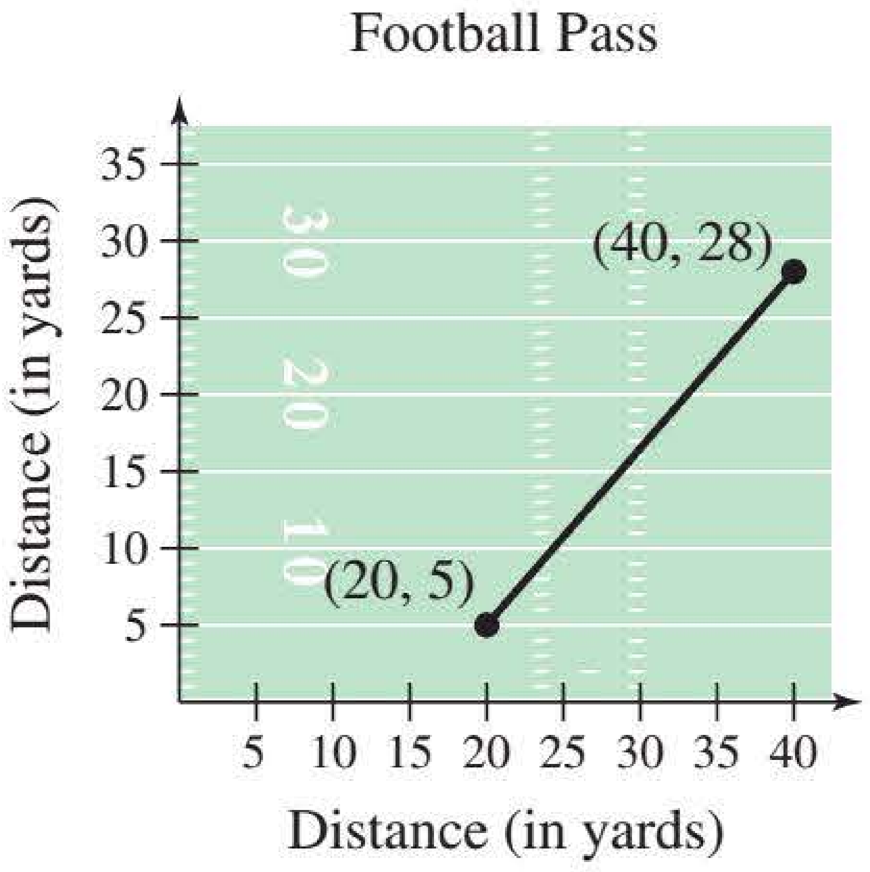

Figure 1.8

Solution You can find the length of the pass by finding the distance between the points (40, 28) and (20, 5).

$$
\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance Formula} \\
&= \sqrt{(40 - 20)^2 + (28 - 5)^2} && \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2. \\
&= \sqrt{20^2 + 23^2} && \text{Simplify.} \\
&= \sqrt{400 + 529} && \text{Simplify.} \\
&= \sqrt{929} && \text{Simplify.} \\
&\approx 30 && \text{Use a calculator.}
\end{aligned}
$$

So, the pass is about 30 yards long.

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A football quarterback throws a pass from the 10-yard line, 10 yards from the sideline. A wide receiver catches the pass on the 32-yard line, 25 yards from the same sideline. How long is the pass?

In Example 6, the scale along the goal line does not normally appear on a football field. However, when you use coordinate geometry to solve real-life problems, you are free to place the coordinate system in any way that is convenient for the solution of the problem.

EXAMPLE 7 Estimating Annual Sales

Starbucks Corporation had annual sales of approximately \$9.8 billion in 2009 and \$11.7 billion in 2011. Without knowing any additional information, what would you estimate the 2010 sales to have been? *(Source: Starbucks Corporation)*

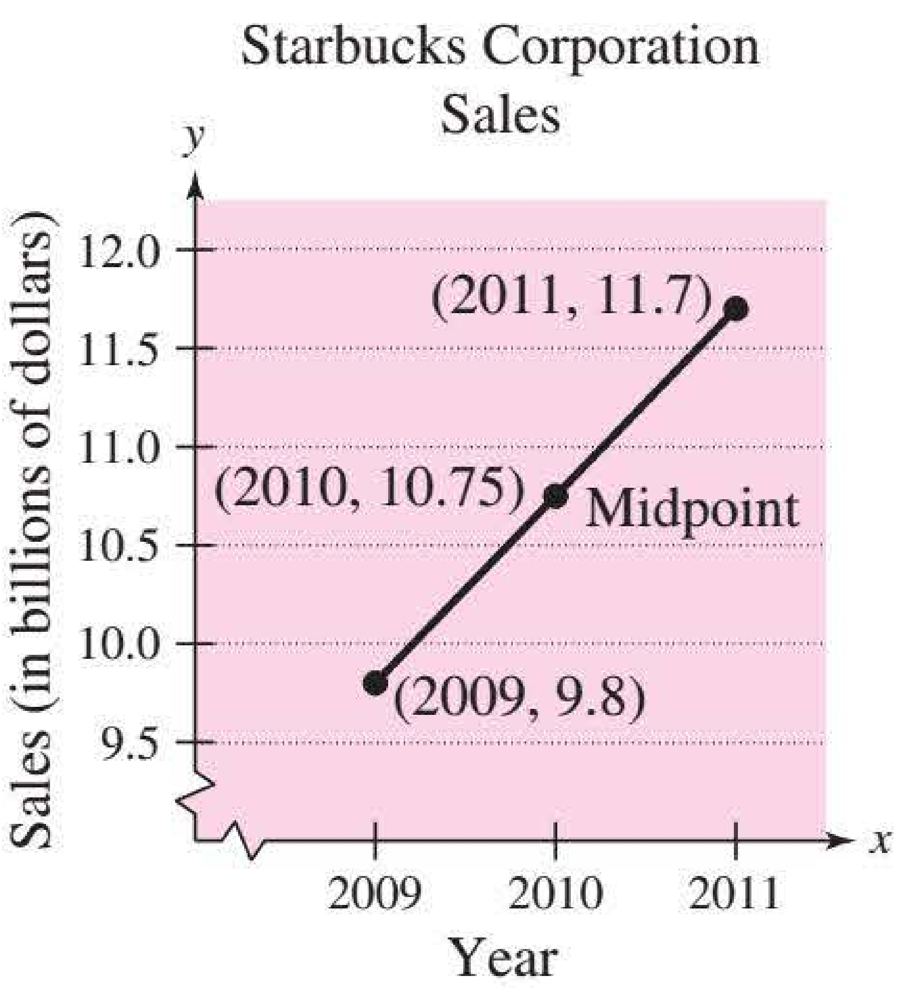

Figure 1.9

Solution One solution to the problem is to assume that sales followed a linear pattern. With this assumption, you can estimate the 2010 sales by finding the midpoint of the line segment connecting the points (2009, 9.8) and (2011, 11.7).

$$
\begin{aligned}
\text{Midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) && \text{Midpoint Formula} \\
&= \left(\frac{2009 + 2011}{2}, \frac{9.8 + 11.7}{2}\right) && \text{Substitute for } x_1, x_2, y_1, \text{ and } y_2. \\
&= (2010, 10.75) && \text{Simplify.}
\end{aligned}
$$

So, you would estimate the 2010 sales to have been about \$10.75 billion, as shown in Figure 1.9. (The actual 2010 sales were about \$10.71 billion.)

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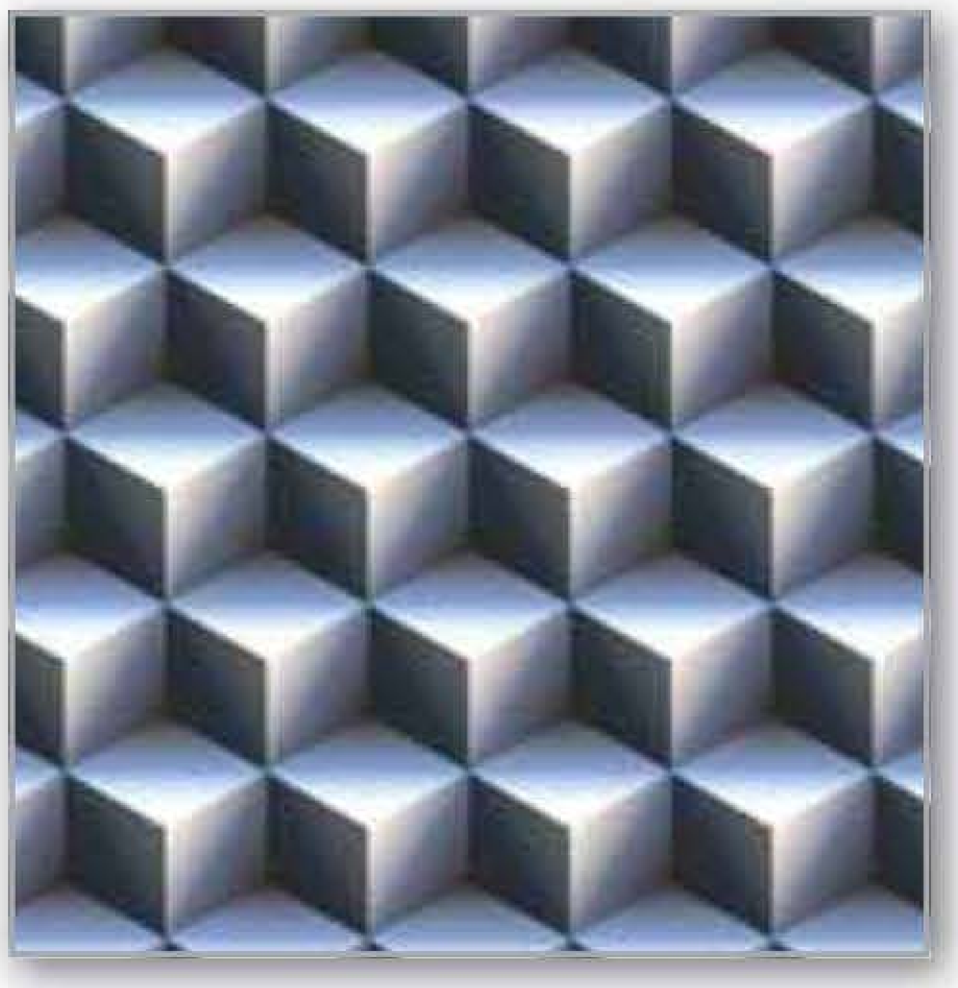
Yahoo! Inc. had annual revenues of approximately \$7.2 billon in 2008 and \$6.3 billion in 2010. Without knowing any additional information, what would you estimate the 2009 revenue to have been? *(Source: Yahoo! Inc.)*



Much of computer graphics, including this computer-generated goldfish tessellation, consists of transformations of points in a coordinate plane. Example 8 illustrates one type of transformation called a translation. Other types include reflections, rotations, and stretches.

EXAMPLE 8 Translating Points in the Plane

The triangle in Figure 1.10 has vertices at the points $(-1, 2)$, $(1, -4)$, and $(2, 3)$. Shift the triangle three units to the right and two units up and find the vertices of the shifted triangle, as shown in Figure 1.11.

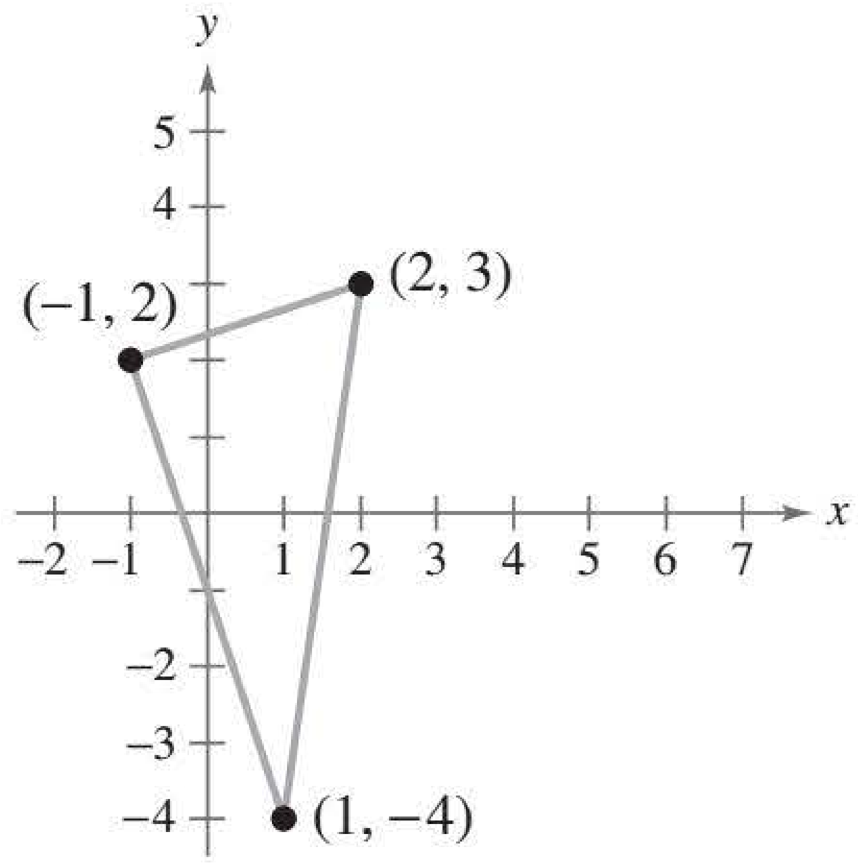


Figure 1.10

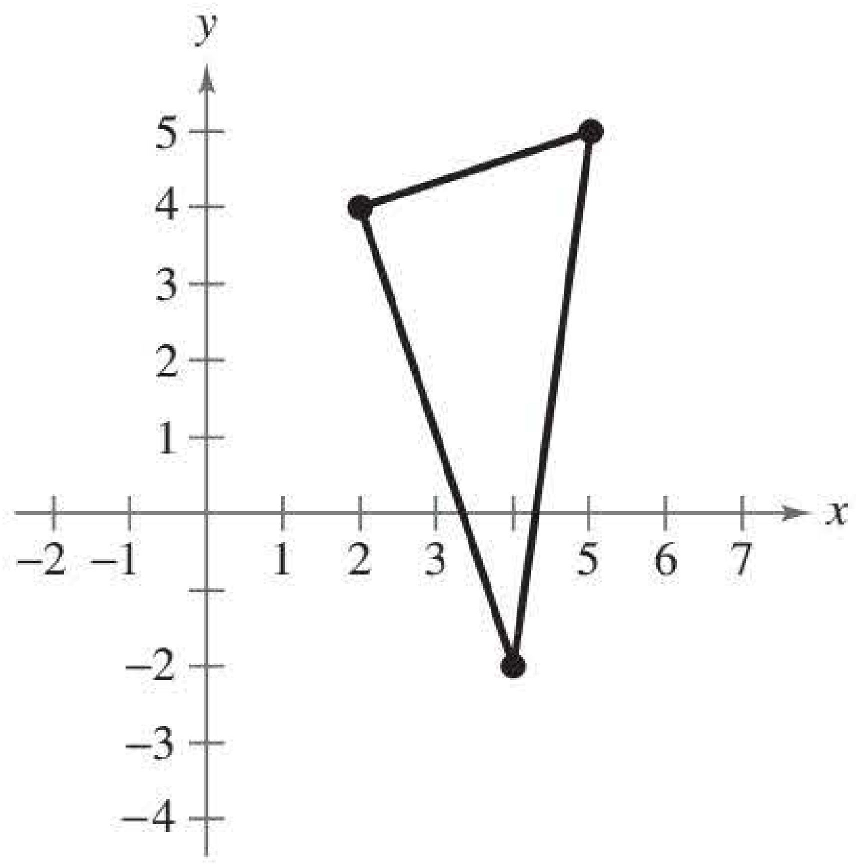


Figure 1.11

Solution To shift the vertices three units to the right, add 3 to each of the x-coordinates. To shift the vertices two units up, add 2 to each of the y-coordinates.

Original Point	Translated Point
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -4)$	$(1 + 3, -4 + 2) = (4, -2)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

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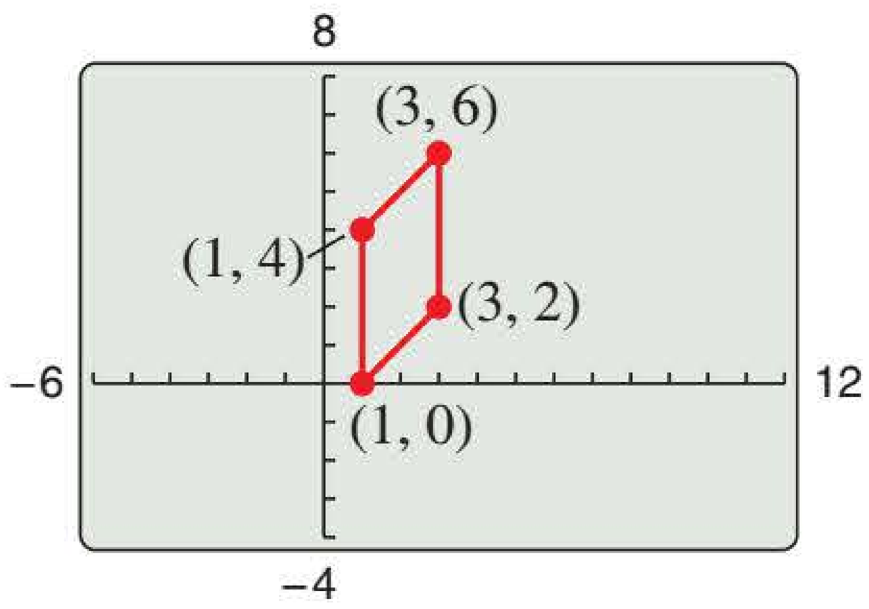
Find the vertices of the parallelogram shown after translating it two units to the left and four units down.



The figures in Example 8 were not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions—even when they are not required.

Summarize (Section 1.1)

1. Describe the Cartesian plane *(page 2)*. For an example of plotting points in the Cartesian plane, see Example 1.
2. State the Distance Formula *(page 4)*. For examples of using the Distance Formula to find the distance between two points, see Examples 3 and 4.
3. State the Midpoint Formula *(page 5)*. For an example of using the Midpoint Formula to find the midpoint of a line segment, see Example 5.
4. Describe examples of how to use a coordinate plane to model and solve real-life problems *(pages 6 and 7, Examples 6–8)*.

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1.1 Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

Vocabulary: Fill in the blanks.

1. An ordered pair of real numbers can be represented in a plane called the rectangular coordinate system or the ________ plane.
2. The point of intersection of the x- and y-axes is the ________, and the two axes divide the coordinate plane into four parts called ________.
3. The ________ ________ is a result derived from the Pythagorean Theorem.
4. Finding the average values of the representative coordinates of the two endpoints of a line segment in a coordinate plane is also known as using the ________ ________.

Skills and Applications

Plotting Points in the Cartesian Plane In Exercises 5 and 6, plot the points in the Cartesian plane.

5. $(-4, 2), (-3, -6), (0, 5), (1, -4), (0, 0), (3, 1)$
6. $\left(1, -\frac{1}{3}\right), (0.5, -1), \left(\frac{3}{7}, 3\right), \left(-\frac{4}{3}, -\frac{3}{7}\right), (-2, 2.5)$

Finding the Coordinates of a Point In Exercises 7 and 8, find the coordinates of the point.

7. The point is located three units to the left of the y-axis and four units above the x-axis.
8. The point is on the x-axis and 12 units to the left of the y-axis.

Determining Quadrant(s) for a Point In Exercises 9–14, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

9. $x > 0$ and $y < 0$
10. $x < 0$ and $y < 0$
11. $x = -4$ and $y > 0$
12. $y < -5$
13. $x < 0$ and $-y > 0$
14. $xy > 0$

Sketching a Scatter Plot In Exercises 15 and 16, sketch a scatter plot of the data shown in the table.

15. The table shows the number y of Wal-Mart stores for each year x from 2003 through 2010. *(Source: Wal-Mart Stores, Inc.)*

DATA

Year, x	Number of Stores, y
2003	4906
2004	5289
2005	6141
2006	6779
2007	7262
2008	7720
2009	8416
2010	8970

Spreadsheet at LarsonPrecalculus.com

16. The table shows the lowest temperature on record y (in degrees Fahrenheit) in Duluth, Minnesota, for each month x, where $x = 1$ represents January. *(Source: NOAA)*

DATA

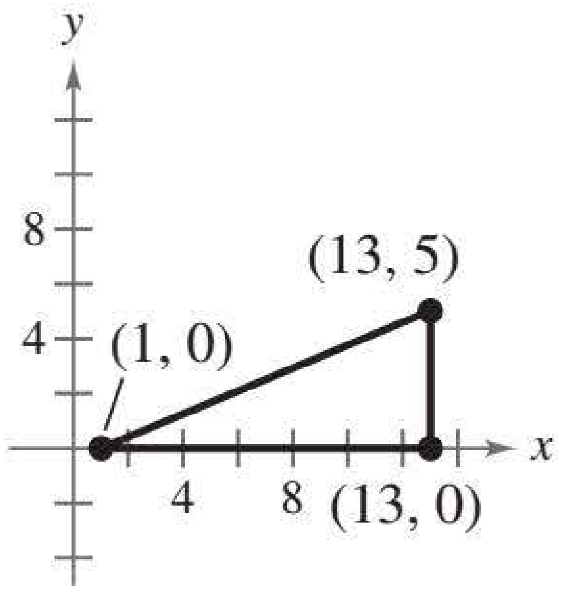
Month, x	Temperature, y
1	−39
2	−39
3	−29
4	−5
5	17
6	27
7	35
8	32
9	22
10	8
11	−23
12	−34

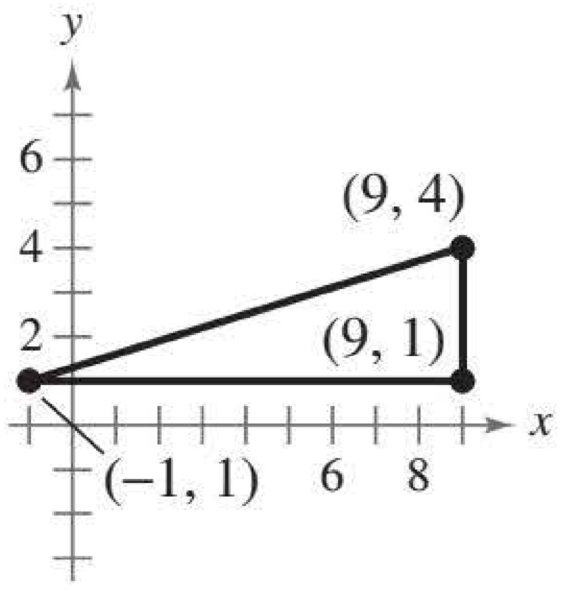
Spreadsheet at LarsonPrecalculus.com

Finding a Distance In Exercises 17–22, find the distance between the points.

17. $(-2, 6), (3, -6)$
18. $(8, 5), (0, 20)$
19. $(1, 4), (-5, -1)$
20. $(1, 3), (3, -2)$
21. $\left(\frac{1}{2}, \frac{4}{3}\right), (2, -1)$
22. $(9.5, -2.6), (-3.9, 8.2)$

Verifying a Right Triangle In Exercises 23 and 24, (a) find the length of each side of the right triangle, and (b) show that these lengths satisfy the Pythagorean Theorem.

23.


24.


Verifying a Polygon **In Exercises 25–28, show that the points form the vertices of the indicated polygon.**

25. Right triangle: $(4, 0), (2, 1), (-1, -5)$

26. Right triangle: $(-1, 3), (3, 5), (5, 1)$

27. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$

28. Isosceles triangle: $(2, 3), (4, 9), (-2, 7)$

Plotting, Distance, and Midpoint **In Exercises 29–36, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.**

29. $(6, -3), (6, 5)$

30. $(1, 4), (8, 4)$

31. $(1, 1), (9, 7)$

32. $(1, 12), (6, 0)$

33. $(-1, 2), (5, 4)$

34. $(2, 10), (10, 2)$

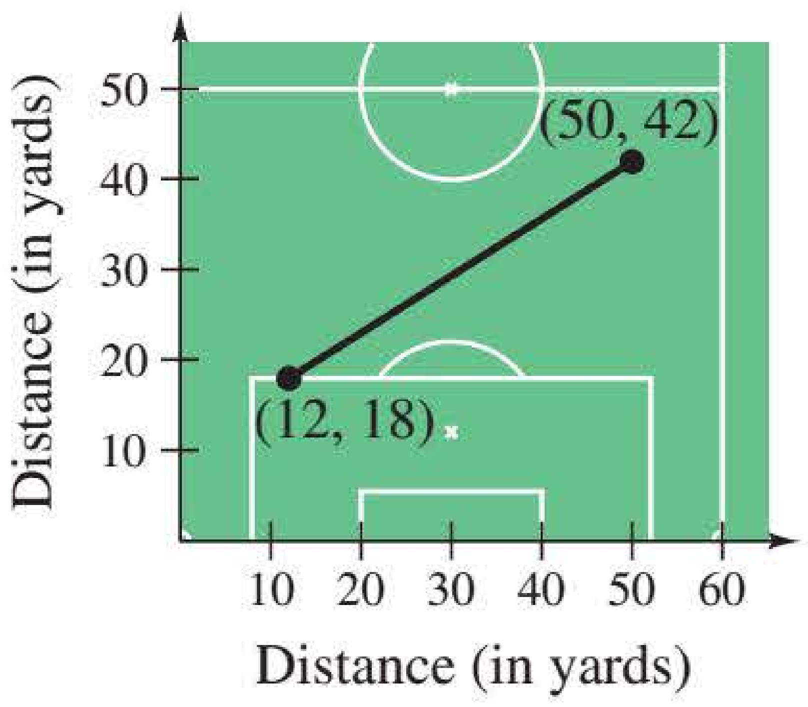
35. $(-16.8, 12.3), (5.6, 4.9)$

36. $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$

37. Flying Distance

An airplane flies from Naples, Italy, in a straight line to Rome, Italy, which is 120 kilometers north and 150 kilometers west of Naples. How far does the plane fly?

38. Sports A soccer player passes the ball from a point that is 18 yards from the endline and 12 yards from the sideline. A teammate who is 42 yards from the same endline and 50 yards from the same sideline receives the pass. (See figure.) How long is the pass?

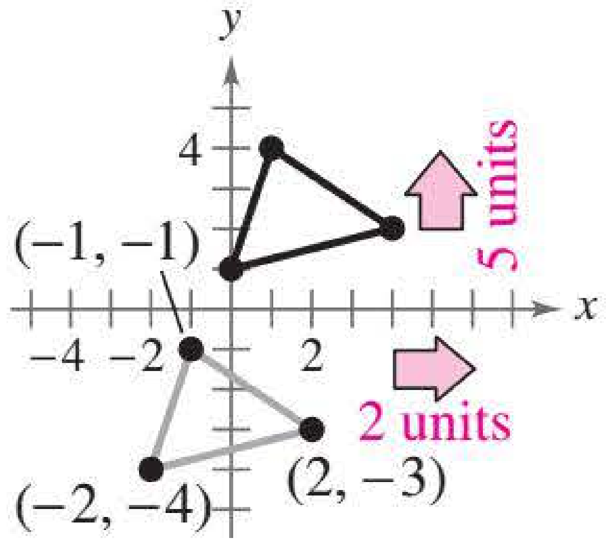


39. Sales The Coca-Cola Company had sales of \$19,564 million in 2002 and \$35,123 million in 2010. Use the Midpoint Formula to estimate the sales in 2006. Assume that the sales followed a linear pattern. *(Source: The Coca-Cola Company)*

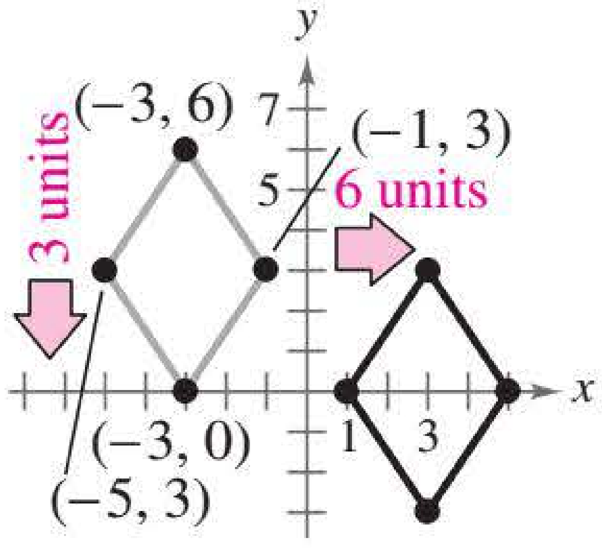
40. Earnings per Share The earnings per share for Big Lots, Inc. were \$1.89 in 2008 and \$2.83 in 2010. Use the Midpoint Formula to estimate the earnings per share in 2009. Assume that the earnings per share followed a linear pattern. *(Source: Big Lots, Inc.)*

Translating Points in the Plane **In Exercises 41–44, find the coordinates of the vertices of the polygon after the indicated translation to a new position in the plane.**

41.



42.



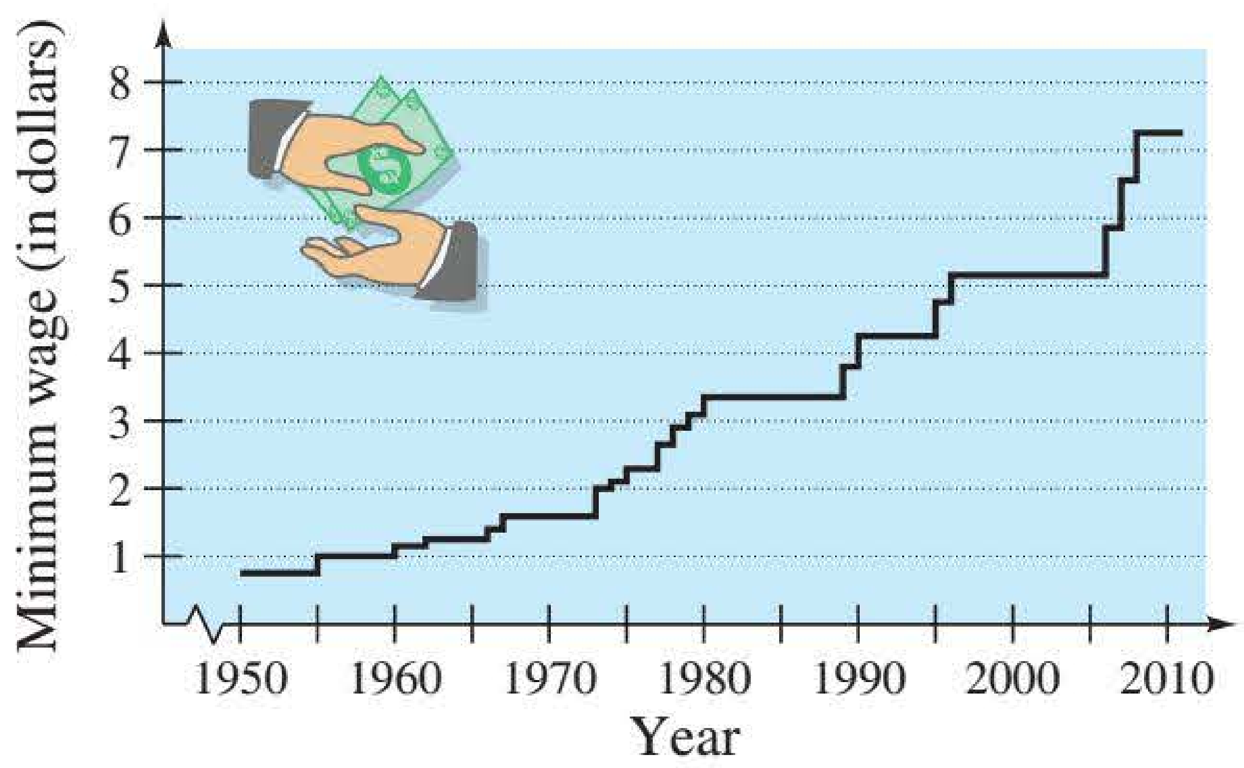
43. Original coordinates of vertices: $(-7, -2), (-2, 2), (-2, -4), (-7, -4)$

Shift: eight units up, four units to the right

44. Original coordinates of vertices: $(5, 8), (3, 6), (7, 6)$

Shift: 6 units down, 10 units to the left

45. Minimum Wage Use the graph below, which shows the minimum wages in the United States (in dollars) from 1950 through 2011. *(Source: U.S. Department of Labor)*



(a) Which decade shows the greatest increase in minimum wage?

(b) Approximate the percent increases in the minimum wage from 1990 to 1995 and from 1995 to 2011.

(c) Use the percent increase from 1995 to 2011 to predict the minimum wage in 2016.

(d) Do you believe that your prediction in part (c) is reasonable? Explain.

46. Data Analysis: Exam Scores The table shows the mathematics entrance test scores x and the final examination scores y in an algebra course for a sample of 10 students.

x	22	29	35	40	44	48	53	58	65	76
y	53	74	57	66	79	90	76	93	83	99

(a) Sketch a scatter plot of the data.

(b) Find the entrance test score of any student with a final exam score in the 80s.

(c) Does a higher entrance test score imply a higher final exam score? Explain.

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Exploration

47. Using the Midpoint Formula A line segment has (x_1, y_1) as one endpoint and (x_m, y_m) as its midpoint. Find the other endpoint (x_2, y_2) of the line segment in terms of x_1, y_1, x_m, and y_m.

48. Using the Midpoint Formula Use the result of Exercise 47 to find the coordinates of the endpoint of a line segment when the coordinates of the other endpoint and midpoint are, respectively,

(a) $(1, -2), (4, -1)$ and (b) $(-5, 11), (2, 4)$.

49. Using the Midpoint Formula Use the Midpoint Formula three times to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four parts.

50. Using the Midpoint Formula Use the result of Exercise 49 to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$ (b) $(-2, -3), (0, 0)$

51. Make a Conjecture Plot the points $(2, 1)$, $(-3, 5)$, and $(7, -3)$ on a rectangular coordinate system. Then change the signs of the indicated coordinates of each point and plot the three new points on the same rectangular coordinate system. Make a conjecture about the location of a point when each of the following occurs.

(a) The sign of the x-coordinate is changed.

(b) The sign of the y-coordinate is changed.

(c) The signs of both the x- and y-coordinates are changed.

52. Collinear Points Three or more points are *collinear* when they all lie on the same line. Use the steps following to determine whether the set of points $\{A(2, 3), B(2, 6), C(6, 3)\}$ and the set of points $\{A(8, 3), B(5, 2), C(2, 1)\}$ are collinear.

(a) For each set of points, use the Distance Formula to find the distances from A to B, from B to C, and from A to C. What relationship exists among these distances for each set of points?

(b) Plot each set of points in the Cartesian plane. Do all the points of either set appear to lie on the same line?

(c) Compare your conclusions from part (a) with the conclusions you made from the graphs in part (b). Make a general statement about how to use the Distance Formula to determine collinearity.

53. Think About It When plotting points on the rectangular coordinate system, is it true that the scales on the x- and y-axes must be the same? Explain.

54. Think About It What is the y-coordinate of any point on the x-axis? What is the x-coordinate of any point on the y-axis?

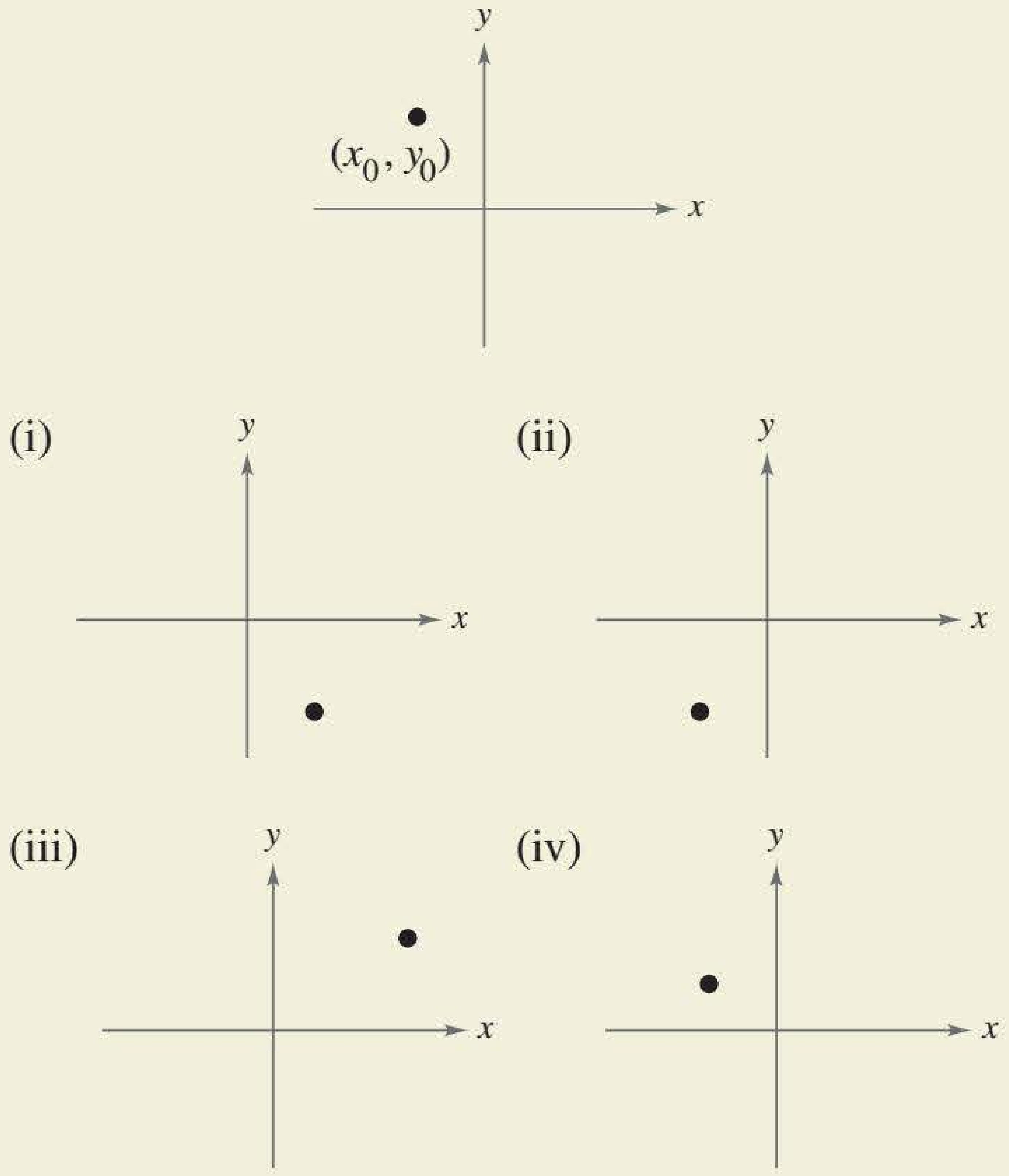
True or False? **In Exercises 55–57, determine whether the statement is true or false. Justify your answer.**

55. In order to divide a line segment into 16 equal parts, you would have to use the Midpoint Formula 16 times.

56. The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.

57. If four points represent the vertices of a polygon, and the four sides are equal, then the polygon must be a square.

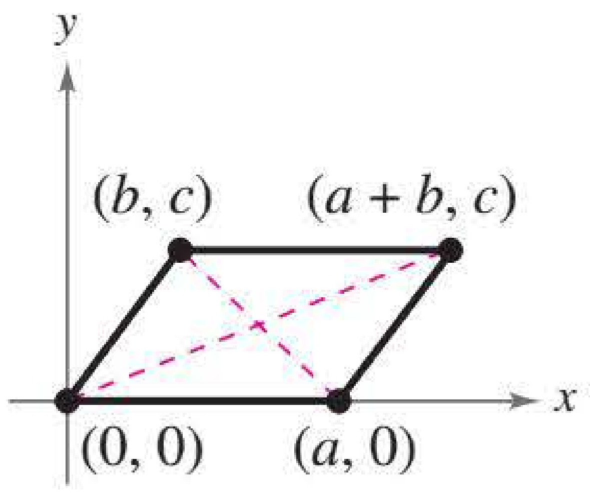
58. HOW DO YOU SEE IT? Use the plot of the point (x_0, y_0) in the figure. Match the transformation of the point with the correct plot. Explain your reasoning. [The plots are labeled (i), (ii), (iii), and (iv).]



(a) (x_0, y_0) (b) $(-2x_0, y_0)$

(c) $(x_0, \frac{1}{2}y_0)$ (d) $(-x_0, -y_0)$

59. Proof Prove that the diagonals of the parallelogram in the figure intersect at their midpoints.



1.2 Graphs of Equations

The graph of an equation can help you see relationships between real-life quantities. For example, in Exercise 87 on page 21, you will use a graph to predict the life expectancy of a child born in 2015.

ALGEBRA HELP When evaluating an expression or an equation, remember to follow the Basic Rules of Algebra. To review these rules, see Appendix A.1.

- **Sketch graphs of equations.**
- **Identify *x*- and *y*-intercepts of graphs of equations.**
- **Use symmetry to sketch graphs of equations.**
- **Write equations of and sketch graphs of circles.**
- **Use graphs of equations in solving real-life problems.**

The Graph of an Equation

In Section 1.1, you used a coordinate system to graphically represent the relationship between two quantities. There, the graphical picture consisted of a collection of points in a coordinate plane.

Frequently, a relationship between two quantities is expressed as an **equation in two variables.** For instance, $y = 7 - 3x$ is an equation in x and y. An ordered pair (a, b) is a **solution** or **solution point** of an equation in x and y when the substitutions $x = a$ and $y = b$ result in a true statement. For instance, $(1, 4)$ is a solution of $y = 7 - 3x$ because $4 = 7 - 3(1)$ is a true statement.

In this section, you will review some basic procedures for sketching the graph of an equation in two variables. The **graph of an equation** is the set of all points that are solutions of the equation.

EXAMPLE 1 Determining Solution Points

Determine whether (a) $(2, 13)$ and (b) $(-1, -3)$ lie on the graph of $y = 10x - 7$.

Solution

a.

$y = 10x - 7$	Write original equation.
$13 \stackrel{?}{=} 10(2) - 7$	Substitute 2 for x and 13 for y.
$13 = 13$	(2, 13) is a solution. ✓

The point $(2, 13)$ *does* lie on the graph of $y = 10x - 7$ because it is a solution point of the equation.

b.

$y = 10x - 7$	Write original equation.
$-3 \stackrel{?}{=} 10(-1) - 7$	Substitute -1 for x and -3 for y.
$-3 \neq -17$	$(-1, -3)$ is not a solution.

The point $(-1, -3)$ *does not* lie on the graph of $y = 10x - 7$ because it is *not* a solution point of the equation.

✓ ***Checkpoint*** *Audio-video solution in English & Spanish at LarsonPrecalculus.com.*

Determine whether (a) $(3, -5)$ and (b) $(-2, 26)$ lie on the graph of $y = 14 - 6x$.

The basic technique used for sketching the graph of an equation is the **point-plotting method.**

The Point-Plotting Method of Graphing

1. When possible, isolate one of the variables.
2. Construct a table of values showing several solution points.
3. Plot these points in a rectangular coordinate system.
4. Connect the points with a smooth curve or line.

John Griffin/The Image Works